Arming in the Global Economy: The Importance of Trade with Enemies and Friends[†]

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Current Version: December 10, 2018

Abstract: We analyze how trade openness matters for interstate conflict over productive resources. Our analysis features a terms-of-trade channel that makes security policies trade-regime dependent. Specifically, trade between two adversaries reduces each one's incentive to arm given the opponent's arming. If these countries have a sufficiently similar mix of initial resource endowments, greater trade openness brings with it a reduction in resources diverted to conflict and thus wasted, as well as the familiar gains from trade. Although a move to trade can otherwise induce greater arming by one of them and thus need not be welfare improving for both, aggregate arming falls. By contrast, when the two adversaries do not trade with each other but instead trade with a third (friendly) country, a move from autarky to trade intensifies conflict between the two adversaries, inducing greater arming. With data from the years surrounding the end of the Cold War, we exploit the contrasting implications of trade between enemies versus trade between friends to provide some evidence that is consistent with the theory.

JEL Classification: D30, D74, F10, F51, F52.
Keywords: resource insecurity, interstate disputes, conflict, arming, trade openness, comparative advantage

[†]This is a substantially revised version of a paper that circulated under the title of "Trading with the Enemy." We are heavily indebted to Stergios Skaperdas for extensive discussions, as well as to Costas Arkolakis and two anonymous referees for extremely useful comments. We also thank Mostafa Beshkar, Rick Bond, Kamal Saggi, Jun Xiang, Ben Zissimos and participants of the ASSA Meetings 2015, the WEAI Annual Conference 2015, the Midwest Economic Theory Conference at Purdue University 2016, the International Economics Workshop at Vanderbilt University and the UC-Irvine/GPACS Workshop on Economic Interdependence and War. Oscar Camacho provided excellent research assistance.

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1 Introduction

International trade takes place within an anarchic setting. Absent an ultimate adjudicator and enforcer, countries inevitably have unresolved disputes and nearly all expend resources on defense to prepare for the possibility of outright conflict or to improve their bargaining positions under the threat of conflict. Despite the anarchic nature of international relations, the classical liberal perspective views greater trade openness among potential adversaries as reducing or even eliminating conflict (e.g., Polachek, 1980). One argument in support of this view is that conflict disrupts trade and thus precludes the realization of at least some of the gains from trade; then, countries acting collectively and wanting to reap those gains would have a greater interest under trade in maintaining a peaceful order. However, it is well-known—with the prisoners' dilemma being a stark and simple example—that in many economic, social and political interactions, collective rationality need not and often does not lead to a stable, equilibrium outcome. Instead, interactions guided by individual rationality often lead to "bad" outcomes. By contrast, emphasizing self-interest and individual rationality as well as the anarchic nature of international relations, the realist/neo-realist perspective argues that trade can aggravate conflict between nations (e.g., Waltz, 1979). Specifically, the benefits from freer trade can fuel frictions and conflict, as some states perceive that they (or their rivals) will develop a military edge in security competition.[1]

Our central objective in this paper is to study how the expansion of international trade affects the intensity of conflict, measured in terms of resources allocated to it. Consider, for example, the ongoing dispute involving China, Taiwan, Vietnam, the Philippines, Indonesia, Malaysia, and Brunei for control over the Spratly and Paracel islands in the South China Sea, where there are oil reserves. Does trade between these rivals pacify their relations inducing them to allocate fewer resources to their respective militaries as might be implied by the classical liberal view, or does it make their rivalry more severe?[2]

Previous theoretical treatments of trade and conflict in the economics literature have identified two distinct channels through which trade between countries could influence their military spending—namely, a factor-price channel and an income channel. For example, based on extensions of Heckscher-Ohlin models that emphasize differences in factor endow-

[1]Pushing this logic one step further, one could argue that actual or potential rivals would not trade with each other (e.g., Grieco, 1990; Gowa, 1995). See Copeland (2015) for a recent survey of the theoretical and empirical literature in international relations regarding trade and conflict.

[2]Similarly, one might ask how globalization in the past few decades has influenced tensions between Russia, Kazakhstan, Turkmenistan, Iran, and Azerbaijan over how to divide the rights of exploration and exploitation for oil in the Caspian Sea. Although these countries came to an agreement in August 2018, many of the details have yet to be worked out, leaving open the possibility that this dispute could continue for some time. (See https://www.nytimes.com/2018/08/12/world/europe/caspian-sea-russia-iran.html and https://www.bbc.com/news/world-45162282.) There are a surprising number of examples where trade has occurred between countries while they were at war with each other—e.g., Standard Oil selling oil to Nazi Germany. (See Barbieri and Levy (1999) and references cited therein.)

ments, Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015) study settings with two small countries that contest a resource and can trade in world markets. A shift from autarky to trade in such settings changes product prices and thus relative factor prices, thereby altering the cost of combining resources in producing military force. Depending on world prices, then, trade can increase or decrease the countries' incentives to arm and thus intensify or pacify their dispute over the resource. Abstracting from this factor-price channel, Garfinkel et al. (2018) study a dynamic model where two countries make consumption, investment and arming decisions in the first period, as they face a strictly positive probability of having to contest a portion of the return from their joint investment in the next period. The effect of trade in the first period is to raise the incomes of both countries and their production of arms, with the initially smaller country gaining some power relative to its larger trading partner and potential foe.

The analysis in the present paper, based on a variant of the Ricardian model of trade suitably extended to allow for international disputes, studies a third channel through which trade can matter for military spending—namely, a terms-of-trade channel. As in the canonical, two-good, two-country Ricardian model, international differences in technology serve as the basis for comparative advantage and provide the rationale for mutually advantageous trade. However, in contrast to the standard Ricardian model, we assume that the input to the production of potentially traded goods is produced with two primary resources, one of which is partially contestable. The contested resource could be oil, minerals, timber, land, or water.[3] Its division depends on the countries' "arms" or "guns," which are also produced domestically using the two primary resources. Since arming is endogenous, so too are the residual resources and the intermediate input used in the production of consumption goods.

A key feature of this setting is the endogeneity of world prices that makes security policies trade-regime dependent.[4] To highlight the importance of this mechanism, we construct a simple model that abstracts from the factor-price and income channels mentioned above.[5] Furthermore, we abstract from many salient features of today's world economy, such as the presence of increasing returns, foreign investment, and endogenous growth. In addition, we do not differentiate between the mobilization of resources for conflict and the potentially destructive deployment of those resources, nor do we consider explicitly the disruptive effect

[3] See Klare (2012), who provides many examples where the competition for scarce resources, for which property rights are not well defined or costlessly enforceable, has turned or can turn violent.

[4] Insofar as trade policies can influence world prices as suggested by the empirical work of Broda, et al. (2008) and Bagwell and Staiger (2011), world prices should depend on resources available to produce traded goods and thus on arming as well. It stands to reason, then, that policymakers take that influence into account when choosing their security (and possibly trade) policies.

[5] Likewise, in the small country settings considered by Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015), the terms-of-trade channel of influence is non-operative. While world prices are endogenous in Garfinkel et al. (2018), they are determined independently of the countries' arming choices. Of course, in more general settings, all three channels of influence could be present.

of conflict to shut down trade between warring nations.[6] Instead, motivated by the empirical relevance of military expenditures, we focus on arming.[7] Insofar as resources are absorbed into the production of arms (thus becoming unavailable for producing goods traded in world markets), these expenditures represent an additional opportunity cost of conflict. Given our focus, this paper can be thought of as an analysis of a modified version of the classical liberal view, one that applies to "cold" wars.

In our setting, we compare the outcomes under two polar trade regimes, autarky and free trade. As one would expect, given the amount of resources allocated to arming, a shift from autarky to free trade unambiguously results in higher payoffs to both countries. However, because in our setting such a switch also influences arming decisions, the welfare consequences of introducing or liberalizing trade can differ significantly from those that typically arise in mainstream analyses that assume perfect and costless security.[8]

Under autarky, where countries can consume only what they produce, each one chooses its arming (or its security policy) so as to equate the marginal benefit of capturing the contested resource to the marginal cost of diverting resources from the country's own production and thus consumption. At the same time, each country's arming choice adversely affects the opponent by reducing its access to the contested resource. In equilibrium, where both countries ignore this negative security externality, arming is strictly positive.

Importantly, trade induces each country to internalize, at least partially, the negative externality of its security policy on the resources available to its rival. The result is to lower arming incentives given the rival's policy. To be more precise, as in the case of autarky, when the two countries trade with each other, each one chooses its arming to balance its marginal benefit with its marginal cost. In the case of trade, however, each country's payoff depends on the production of its adversary's exported good, which the adversary produces relatively more efficiently. Accordingly, an increase in one's own arms has an additional cost under trade: a reduction in the adversary's share of the contested resource and thus a reduction in the adversary's production of its exports. The added cost, which is reflected in a deterioration of the importing country's terms of trade that lowers the marginal benefit

[6]This is not to deny the importance of conflict's disruptive effect on trade. To the contrary, this effect is empirically relevant (Glick and Taylor, 2010), and can be viewed as an opportunity cost of conflict that serves as the basis for classical liberal view as suggested above. In related research, we explore the adversarial countries' arming choices and their subsequent decision to either fight that would preclude trade between them or negotiate a peaceful settlement (Garfinkel and Syropoulos, 2018b).

[7]Researchers from the Stockholm Institutional Peace Research Institute (SIPRI) estimate that, in 2017, global military expenditures were $1,739 billion, accounting for 2.2 percent of global GDP. The five biggest spenders that year (in current U.S. dollar terms) were the United States ($610 billion or 3.1 percent of GDP), China ($228 billion or 1.9 percent), Saudi Arabia ($69.4 billion or 10 percent), Russia ($66.3 billion or 4.3 percent), and India ($63.9 billion or 2.5 percent). See Tian et al. (2018) for more details.

[8]Nevertheless, our welfare analysis and comparison of trade regimes resemble those in Arkolakis et al. (2012) and Costinot and Rodríguez-Clare (2014). The primary difference is that, in our work, real income is endogenously determined under non-cooperative interactions in arming.

of arming relative to the marginal cost, means that a country's incentive to arm, given the adversary's arming choice, is strictly lower under free trade than under autarky. This finding is robust to the presence of trade costs and does not disappear when countries simultaneously and non-cooperatively choose tariffs.

Of course, a shift from autarky to trade on *equilibrium* arming and welfare depends not only on the direct effect that can be viewed as an inward shift in each country's best-response function, but also on the indirect or strategic effect as the rival country's arming falls. However, we find that the strategic effect is, by and large, of second-order importance. That is to say, provided the distribution of the contested resource (what we call "capital") and the uncontested resource (what we call "labor") across the adversarial countries is not severely uneven, the strategic effect could reinforce the direct effect or, even if it moves in the opposite direction, is swamped by the direct effect. In such cases, since trade is no worse than autarky for a given level of arming, the reduction in arming and thus security costs due to trade render trade unambiguously superior to autarky. This added benefit, not captured by mainstream trade theory that abstracts from the insecurity of property, is consistent with the spirit of classical liberalism and the writings of authors such as Angell (1933) who extolled the virtues of trade openness and globalization.

But, there do exist sufficiently asymmetric distributions of labor and capital resources such that one country—specifically, the country whose capital-to-labor ratio is sharply larger to imply a sufficiently higher opportunity cost of arming under autarky than its rival—is induced to increase its arms as its opponent reduces its arming in response to a shift from autarky to free trade.[9] Even though the country that is induced to arm by more remains less powerful under trade, the resulting adverse strategic effect realized by the rival could swamp its gains from trade to render trade unappealing. This finding is reminiscent of the realist/neorealist view in the international relations literature (mentioned above) that highlights trade's effect to generate uneven gains to trading partners and thereby differentially influence their arming and thus the balance of power.[10] However, in our analysis, the differential influence of trade on the two countries' arming choices and the implied influence on the balance of power hinges on sharp differences in the mix of their initial holdings of secure capital and labor, not simply on differences in the size of their economies.[11] Nonetheless, even in such cases, we find that a move to trade, whether costly

[9]This possibility is consistent with the empirical finding of Morelli and Sonno (2017) that asymmetries in oil resource endowments across two countries reduce the pacifying effect of bilateral dependence between them in trade and with Beviá and Corchón's (2010) theoretical finding that sharp asymmetries in endowments can reduce the effectiveness of resource transfers between countries to avoid war.

[10]Also see Garfinkel et al. (2018) mentioned above.

[11]Also see Bonfatti and O'Rourke (2018), who consider the role of increasing trade dependence for two adversarial countries with the rest of the world, in a dynamic, leader-follower setting, to influence the likelihood of a preemptive war by the follower. In that analysis, similar to ours, understanding the emergence of conflict between the two countries does not hinge on differences in the size of their economies. But, in

or not, results in lower aggregate arming.[12]

The logic of the terms-of-trade channel has sharply different implications for the effect of increased integration of world markets on the incentives to arm by adversarial countries that do not directly trade with each other, but instead with a friendly country. To isolate these differences, we focus on adversaries that have a similar comparative advantage.[13] Suppose, in particular, that the two adversarial countries are identical in every respect so that, even in the absence of barriers to trade, they would not trade with each other. Also, suppose that there exist technological differences between these countries on the one hand and the third country on the other hand that, given the resources allocated to arming, make trade mutually advantageous. In this case where the two adversaries compete in the market for the same good exported to a third country, there is an added marginal benefit to arming under trade—namely, a positive terms-of-trade effect—implying increased arming incentives under trade relative to those under autarky. That expanded trade opportunities with another (friendly) country can intensify conflict between two adversaries is similar in spirit to Martin et al. (2008), who show that increasing the opportunities for trade among all countries reduces the interdependency between any two and thus can make conflict between them more likely. However, while that analysis emphasizes the importance of the disruptive effects of conflict, ours highlights the importance of the endogeneity of arming and its trade-regime dependence. Furthermore, our analysis, like Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015), suggests that, in this case, trade between friends could make the adversaries worse off.[14]

Based on the different implications of trade with the enemy versus trade with friends for arming choices, we also provide some suggestive evidence in support of the theory. Our empirical analysis focuses on the influence of trade openness, measured by trade volumes, on national military spending with data surrounding the end of the Cold War. A key feature of this analysis is that it differentiates between countries that have no rivals and countries that have rivals, using data on bilateral "strategic rivalries" from Thompson (2001); for countries

Bonfatti and O'Rourke's analysis, one fundamental sort of asymmetry is critical—that is, the leader's ability to block imports (necessary for arming) to the rival.

[12]Acemoglu and Yared (2010) find empirically that military expenditures and the size of the military in terms of personnel are negatively related to trade volumes. Although that analysis treats the military variables (reflecting the "nationalist" or "militarist" sentiments) as exogenous, the negative relationship found can be viewed as preliminary evidence in support of the modified version of the classical liberal view we consider here, focusing on military expenditures. In any case, as discussed below, we provide additional evidence in support of this view.

[13]Consider, for example, India and Pakistan and their ongoing conflict over Kashmir.

[14]It is important to emphasize, though, that the result in this paper is due to the beneficial effect that a country's arming has on its own terms of trade to add to arming incentives, whereas the result in Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015) derives from the impact of world prices to increase arming incentives through factor prices. A similar result arises in settings with conflict over some resource between groups within a single (small) country that trades with the rest of the world (e.g., Garfinkel et al., 2008).

that do have rivals, we further differentiate between their trade with rivals and their trade with friends. Our main finding is that the distinction is statistically significant, with the qualitative differences as predicted. Specifically, the estimates confirm that a country's military spending is inversely related to its trade with rivals, but positively related to its trade with friends. These results, which are robust to a series of alternative specifications that address concerns of possible endogeneity of the trade variables in the econometric model, complement the more structural evidence provided by Martin et al. (2008) that increased trade need not always promote peace and by Seitz et al. (2015) that reductions in trade costs bring added benefits largely through their effect to dampen defense spending, not only by trading partners, but other countries as well.

In what follows, the next section presents the basic model, focusing on just two countries that trade with one another. Section 3 derives the countries' payoff functions under the regimes of autarky and free trade In Section 4, we characterize the countries' incentives to arm under each trade regime. Section 5 studies how equilibrium arming and payoffs compare across trade regimes and discusses how the central results remain intact with the introduction of trade costs. In Section 6, we extend the framework by introducing a third (non-adversarial) country. In Section 7, we present our empirical evidence. Concluding remarks follow in Section 8. Technical and supplementary details are relegated to appendices.

2 Contesting a Resource in a Ricardian Setting

Consider a world with two countries, indexed by a superscript $i = 1, 2$. Each country i holds secure endowments of two productive resources: labor denoted by L^i and capital denoted by K^i. Capital could be land, oil, minerals, timber or water resources. In contrast to standard trade models, we suppose that there is an additional amount of capital, denoted by K_0, that is insecure and contestable by the two countries.[15]

In an effort to contest K_0, each country i produces guns G^i $(i = 1, 2)$, which we view as a composite good reflecting the country's military strength. Specifically, the share of K_0 secured by country i is given by

$$\phi^i = \phi(G^i, G^j) = \frac{f(G^i)}{f(G^i) + f(G^j)}, \quad i \neq j = 1, 2, \tag{1}$$

where $f(\cdot) > 0$, $f(0)$ is arbitrarily close to 0, $f'(\cdot) > 0$, and $f''(\cdot) \leq 0$. This specification of the conflict technology, also known as the "contest success function," is symmetric so that $G^1 = G^2 \geq 0$ implies $\phi^1 = \phi^2 = \frac{1}{2}$; it also implies ϕ^i is increasing in country i's own guns

[15] With an appropriate choice of K_0 and K^i, our analysis could capture conflicts over what would appear to be one country's resource. See Caselli et al. (2015), who examine empirically the importance of the proximity of oil fields to countries' shared border in the escalation of conflict between them.

($\phi^i_{G^i} > 0$) and decreasing in the guns of its adversary ($\phi^i_{G^j} < 0$, $j \neq i$).[16] The influence of guns on the division of K_0 between the two countries can be thought of as the result of either open conflict (without destruction) or a bargaining process with the countries' relative military strength playing a prominent role.[17]

Guns are produced with *secure* labor and/or capital endowments. Letting w^i and r^i be the competitive rewards paid to labor and capital in country i respectively, define $\psi(w^i, r^i)$ as the cost of producing one gun in country i. This unit cost function, which is identical across countries, is increasing, concave and homogeneous of degree one in factor prices. By Shephard's lemma, its partial derivatives ψ^i_w and ψ^i_r give the conditional demands respectively for labor and capital in the production of one gun. Thus, the quantities of labor and capital diverted to contesting K_0 in each country i are respectively $\psi^i_w G^i$ and $\psi^i_r G^i$, with ψ^i_r/ψ^i_w representing the corresponding capital-labor ratio. Throughout, we assume that the secure labor and capital resource constraints do not bind in the production of guns for either country i: $L^i - \psi^i_w G^i > 0$ and $K^i - \psi^i_r G^i > 0$.

Once guns have been produced and the disputed resource has been divided, the residual quantities of labor and capital available to country i are respectively $L^i_Z = L^i_g - \psi^i_w G^i$ and $K^i_Z = K^i_g - \psi^i_r G^i$, where $L^i_g \equiv L^i$ and $K^i_g \equiv K^i + \phi^i K_0$ denote "gross" quantities of these primary resource factors. Each country i combines its own residual resources, L^i_Z and K^i_Z, to produce an intermediate input, Z^i. Both factors are essential, and the technology is described by the unit cost function $c(w^i, r^i)$. This function, like that for guns, is identical across countries, and increasing, concave and linearly homogeneous in factor prices. Furthermore, c^i_w and c^i_r give the conditional demands respectively for labor and capital in the production of a unit of Z^i, with c^i_r/c^i_w indicating the capital-labor ratio demanded in that sector of the economy.

The intermediate good Z^i, in turn, can be used to produce two goods for final consumption, X^i_j ($j = 1, 2$).[18] Suppose, in particular, that to produce one unit of good j ($= 1, 2$) producers in country i ($= 1, 2$) need $a^i_j > 0$ units of Z^i. Thus, we have the following

[16]See Skaperdas (1996), who axiomatizes a similar specification that is standard in the contest and conflict literatures. Our slight modification, requiring $f(0)$ to be positive but arbitrarily close to 0, is helpful in our proofs of existence of equilibrium in arming choices under the trade regimes studied. (One example is $f(G) = (\delta + G)^b$ with $b \in (0, 1]$ and $\delta > 0$.) More generally, the appeal of this specification for our analysis derives from the fact that, because it is well understood, it allows us to highlight the effects of the option to trade for arming incentives.

[17]See Anbarci et al. (2002) who study how alternative bargaining solution concepts translate into rules of division that differ in their sensitivity to guns. Also see Garfinkel and Syropoulos (2018a) for a related analysis in a trade-theoretic setting.

[18]With a focus on just two consumption goods, our analysis considers adjustments only at the intensive margin, but could be extended to consider a continuum of consumption goods, as in Dornbusch et al. (1977). We chose the two-good version of the model in view of researchers' general familiarity with it and because it reveals the possibility that trade in the presence of conflict can be welfare reducing, which does not seem to arise when there is a continuum of goods.

production-possibilities constraint:

$$a_1^i X_1^i + a_2^i X_2^i = Z^i, \text{ where } X_j^i \geq 0. \tag{2}$$

Observe from (2) that, as in the canonical 2-country, 2-good, one-factor Ricardian trade model, comparative advantage is driven by international differences in technology (i.e., productivity) and not by differences in factor endowments.[19] To fix ideas, we assume that country i has a comparative advantage in good i. That is, $a_i^i/a_j^i < a_i^j/a_j^j$, which states that the opportunity cost in country i to produce good i $(= 1, 2)$ is lower than the corresponding opportunity cost in country j $(\neq i)$. To economize on notation and emphasize the importance of other variables of interest, we normalize a_i^i $(= 1)$ for $i = 1, 2$ and let $\alpha^i = a_j^i > 1$ for $i \neq j = 1, 2$. Note, in any case, this model differs from the Ricardian trade model in a significant way. Specifically, each country i's production of Z^i (and thus each country's productive potential and income) depends on the security policies of both countries (G^i for $i = 1, 2$), which are endogenous.

Bringing the essential elements of the model together, the sequence of events is as follows:

Stage 1. The two countries $(i = 1, 2)$ simultaneously choose their guns G^i using their secure endowments of labor and capital.

Stage 2. The contested resource K_0 is divided according to the ϕ^i combination induced by these choices. Each country i then uses all its available inputs ($L_Z^i = L_g^i - \psi_w^i G^i$ units of labor and $K_Z^i = K_g^i - \psi_r^i G^i$ units of capital) to produce the intermediate input, Z^i.

Stage 3. Each country i uses its output of Z^i to produce X_i^i and X_j^i units of consumption goods i and j respectively, which are traded domestically and/or internationally depending on the trade regime in place.

In the two subsections to follow, we complete the specification of the model and present some preliminary analysis related to stages 2 and 3 respectively. This preliminary analysis allows us to subsequently analyze the equilibrium in security policies chosen in stage 1, contingent on the trade regime in place.

2.1 Factor Markets and Production of the Intermediate Input

Whether the two countries trade or not, perfect competition requires the price of the intermediate input (p_Z^i) to be equal to its corresponding marginal cost (c^i); as such, we use

[19] We impose this production structure with an intermediate input for convenience only. An analytically equivalent approach would be to assume that the production functions of traded goods in each country differ by a Hicks neutral factor of proportionality, but not in factor intensities. Abstracting from differences in factor intensities allows us to rule out factor-endowment based rationales for trade patterns and the possible adjustments in arming decisions due to trade related factor-price effects (as in extensions of the Heckscher-Ohlin model) and focus solely on terms-of-trade effects.

these concepts interchangeably. Additionally, the amount of each factor used to produce Z^i and G^i must equal its gross supply:

$$c_r^i Z^i + \psi_r^i G^i = K_g^i \ (\equiv K^i + \phi^i K_0) \tag{3a}$$

$$c_w^i Z^i + \psi_w^i G^i = L_g^i \ (\equiv L^i), \tag{3b}$$

for $i = 1, 2$. Now let $k_Z^i \equiv K_Z^i / L_Z^i$ denote country i' residual capital-labor ratio given G^i and G^j. Then, combining (3a) and (3b) gives

$$\frac{c_r^i}{c_w^i} = k_Z^i \equiv \frac{K_g^i - \psi_r^i G^i}{L_g^i - \psi_w^i G^i}, \tag{4}$$

which implicitly defines the equilibrium wage-rental ratio $\omega^i \equiv w^i / r^i$ in country i as a function of K_g^i, L_g^i, and G^i to ensure factor-market clearing—that is, $\omega^{ie} \equiv \omega^{ie}(K_g^i, L_g^i, G^i)$.

Turning to the value of each country's production of the intermediate input denoted by $R^i = c^i Z^i$, (3a) and (3b) with the linear homogeneity of the unit cost functions imply

$$R^i = (w^i c_w^i + r^i c_r^i) Z^i = w^i L_Z^i + r^i K_Z^i = w^i L_g^i + r^i K_g^i - \psi^i G^i,$$

from which we obtain:

$$Z^i(\omega^i, K_g^i, L_g^i, G^i) = \frac{w^i L_g^i + r^i K_g^i - \psi(w^i, r^i) G^i}{c(w^i, r^i)} = \frac{\omega^i L_g^i + K_g^i - \psi(\omega^i, 1) G^i}{c(\omega^i, 1)}. \tag{5}$$

Then, using ω^{ie} implied by (4) in the right-hand side (RHS) of (5) gives $Z^{ie} \equiv Z^{ie}(K_g^i, L_g^i, G^i)$, which we refer to as the optimized production of the intermediate input. Importantly, given G^i and thus K_g^i and L_g^i, this optimized quantity is independent of the trade regime in place.[20] As will become obvious below, this function plays a pivotal role in the definition of the countries' payoff functions and thus in our analysis of equilibrium security policies. Henceforth, to avoid notational cluttering, we omit the "e" designation in the superscript when referring to the equilibrium values of Z^i and ω^i.

2.2 Preferences and Product Demand

Consumer preferences in each country i are given by $U^i = U(D_i^i, D_j^i)$ for $j \neq i = 1, 2$, where D_j^i denotes the quantity of consumption good j $(= 1, 2)$ demanded in country i $(= 1, 2)$. This utility function is increasing, quasi-concave, homogeneous of degree one and symmetric over the two goods, with a constant and finite elasticity of substitution, denoted in absolute

[20]See Lemma A.1 in the Appendix. This lemma also shows how Z^{ie} and ω^{ie} depend on L_g^i, K_g^i and G^i. While equation (5) might appear to be a regular production function, the expression is best thought of as showing the equilibrium quantity Z^{ie} that is jointly determined with factor prices and with gross factor inputs K_g^i and L_g^i being employed (possibly) in different proportions than in the production of G^i.

terms by $\sigma \in (0, \infty)$.[21]

Now, let p^i_j denote the price of consumption good j $(= 1, 2)$ in country i $(= 1, 2)$. To keep the analysis as transparent as possible, we abstract for now from trade costs.[22] Competitive pricing in product markets always requires $p^i_i = c^i$, since country i always produces the good in which it has a comparative advantage (good i). Furthermore, absent trade costs, perfect competition requires that the domestic price of the good in which country i has a comparative disadvantage satisfy $p^i_j = \min[\alpha^i c^i, c^j]$ for $i \neq j = 1, 2$. When trade is possible, at least one country will cease to produce that good; in this case $p^i_j = p^j_j = c^j$ for $i \neq j = 1$ and/or 2. But, if trade is simply ruled out, then $p^i_j = \alpha^i c^i$.

Next, let Y^i denote country i's national income. Under both autarky and trade (absent tariffs), we have $Y^i = R^i = c^i Z^i$. Then, country i's indirect utility function can be written as $V^i = \mu^i Y^i = \mu^i c^i Z^i$, where $\mu^i = [(p^i_i)^{1-\sigma} + (p^i_j)^{1-\sigma}]^{1/(\sigma-1)}$ for $i \neq j = 1, 2$ represents the marginal utility of income, which is decreasing and homogeneous of degree -1 in prices.[23] By Roy's identity, the demand function for good j in country i is

$$D^i_j = \gamma^i_j Y^i / p^i_j = \gamma^i_j c^i Z^i / p^i_j, \tag{6}$$

where $\gamma^i_j \equiv \gamma^i_j(p^i_i, p^i_j) = -(\partial\mu^i/\partial p^i_j)/(\mu^i/p^i_j) = (p^i_j)^{1-\sigma}/[(p^i_j)^{1-\sigma} + (p^i_i)^{1-\sigma}]$ is the expenditure share on good j in country i. Naturally, the relative demand $RD^i = D^i_j/D^i_i$ for good j in country i depends solely on relative prices. Specifically, define $p^i \equiv p^i_j/p^i_i$ as the price of good j in country i relative to the good in which country i enjoys a comparative advantage, and let a hat "$\hat{}$" denote percentage change (e.g., $\widehat{x} \equiv dx/x$). Then, we have $\widehat{RD}^i = -\sigma\widehat{p}^i$.

3 Trade Regimes and Payoff Functions

To explore how the countries' arming incentives depend on the trade regime in place—namely, autarky and free trade—we first analyze how their payoffs are determined under each of these regimes for given security policies, G^i, and gross endowments, K^i_g and L^i_g.

3.1 Autarky

We start our analysis of a closed economy with two observations. First, self-sufficiency requires both consumption goods to be produced in each country i; therefore, $p^i_i = c^i$ and

[21] Assuming CES preferences is not critical here, but helps simplify notation. Our assumption that the consumption goods are imperfect substitutes ($\sigma < \infty$) eases the exposition. As will become clear, when $\sigma = \infty$, the trade-dependency of arming incentives disappears.

[22] We explicitly consider trade costs in Section 5.3 and Supplementary Appendix B.2.

[23] More precisely, μ^i is the inverse of the price index that is dual to the CES aggregator. Observe the linear homogeneity of utility implies risk neutrality; however, due to the concavity of the technology for Z^i in capital, this assumption does not imply the equivalence of the contest we are considering and a "winner-take-all contest," with ϕ^i equaling country i's probability of taking control of the entire quantity of the disputed resource. As can easily be confirmed, under autarky given their guns choices, the two countries strictly prefer (*ex ante*) to "share" the disputed resource according to (1).

$p^i_j = \alpha^i c^i$, which give the equilibrium relative price under autarky: $p^i_A = p^i_j/p^i_i = \alpha^i$. Second, with the demand functions (6), this relative price determines the division of income between them. This division along with the product market-clearing requirement that $D^i_j = X^i_j$ for each good j determines (by (2)) equilibrium quantities.

Using the competitive pricing relations noted above in $\mu(p^i_i, p^i_j)$ and the fact that $Y^i = R^i = c^i Z^i$, country i's payoff function under autarky can be written as

$$V^i_A = \mu(c^i, \alpha^i c^i) Y^i = m^i_A Z^i(K^i + \phi^i K_0, L^i, G^i), \quad i = 1, 2, \tag{7}$$

where $m^i_A \equiv \mu(c^i, \alpha^i c^i) c^i = \mu(1, p^i_A)$ and $Z^i \equiv Z^i(\omega^i, \cdot)$ satisfies (4) and (5). An important feature of country i's payoff function under autarky (7) is that, since $p^i_A = \alpha^i$ is constant, so is m^i_A. Thus, each country i's arming decision under autarky depends solely on how G^i influences the optimized value of the intermediate input, Z^i.

3.2 Free Trade

Before examining arming incentives under autarky, we characterize the equilibrium payoff functions given guns under trade. We focus on the case where country i's demand for good j is fulfilled entirely by imports, and the equilibrium determination of world prices that clears the market for good j satisfies $D^i_j + D^j_j = X^j_j = Z^j$ for $j \ (\neq i) \ = 1, 2$.

To proceed, recall that competitive forces under free trade imply consumers face identical prices for each good: $p^i_j = p^j_j = c^j$ for $j = 1, 2$. Since consumer preferences are identical across countries, arbitrage implies that their expenditure shares on that good are identical: $\gamma^i_j = \gamma^j_j = \gamma_j$ for $j \neq i = 1, 2$. Maintaining our focus on the market for good j, now let p^i_T ($\equiv p^j_j/p^j_i = c^j/c^i$) denote the relative world (or domestic) price of country i's imported good ($j \neq i$). Substitution of the demand functions (6) with the competitive pricing relations into the relevant market-clearing condition shows

$$p^i_T = \gamma_j Z^i / \gamma_i Z^j, \qquad i \neq j = 1, 2, \tag{8}$$

where now $\gamma_j = (p^i_T)^{1-\sigma}/[1 + (p^i_T)^{1-\sigma}]$ and $\gamma_i = 1 - \gamma_j$.[24]

One can identify the impact of the relative supply of the countries' intermediate inputs on p^i_T by logarithmically differentiating (8), while accounting for the dependence of the expenditure shares on p^i_T. After rearranging terms, we have

$$\widehat{p}^i_T = \frac{1}{\sigma}\left[\widehat{Z}^i - \widehat{Z}^j\right], \qquad i \neq j = 1, 2. \tag{9}$$

An exogenous increase in the relative supply of country i's intermediate input Z^i/Z^j expands

[24] Of course, depending on the countries' security policies, factor endowments, technology, and consumer preferences, p^i_T could be determined by one country's autarkic price. We revisit this possibility below.

the relative supply of its exported good and thus worsens its terms of trade p_T^i.

Since $Y^i = c^i Z^i$, we write country i's payoff function under trade as:

$$V_T^i = \mu(p_i^i, p_j^i) Y^i = m_T^i Z^i(K^i + \phi^i K_0, L^i, G^i), \quad i \neq j = 1, 2, \tag{10}$$

where $m_T^i \equiv \mu(p_i^i, p_j^i) c^i = \mu(1, p_T^i)$ and once again $Z^i \equiv Z^i(\omega^i, \cdot)$ satisfies (4) and (5). Guns production influences the payoffs under trade, like the payoffs under autarky, through its effect on the maximized value of the country's intermediate output Z^i. However, country i's production of guns influences V_T^i through an additional channel—namely, through its effect on the relative price of country i's imported good, p_T^i. Specifically, from (9), an increase in G^i, given G^j, increases Z^i and decreases Z^j, and thereby increases the relative price of country i's imported good, p_T^i. Since $\frac{\partial m_T^i}{\partial p_T^i} \frac{p_T^i}{m_T^i} = -\gamma_j < 0$, equation (10) implies an increase in p_T^i alone reduces country i's payoff under trade V_T^i.

3.3 Incentives to Trade (for Given Guns)

While one of our primary goals is to explore the trade-regime dependency of arming incentives and the associated welfare implications, it is instructive to see how payoffs under autarky and trade compare for given guns. Based on a standard, gains-from-trade argument using (7) and (10), one can verify the following:

Lemma 1 *For any given feasible guns and gross factor endowments, payoffs under autarky and trade are ranked as follows: $V_A^i \leq V_T^i$, for each $i = 1, 2$.*

Intuitively, when country i's cost of importing good j (p_T^i) is greater than its opportunity cost $\alpha^i = p_A^i$, country i produces both goods locally, implying identical payoffs under the two trade regimes. Trade flows between the two countries will be strictly positive, given guns, only when $p_T^i \leq \alpha^i$ with strict inequality for at least one country to make the payoffs for both countries under trade at least as high as they are under autarky and strictly greater under trade for at least one of them.[25] Any added payoff generated by a shift to free trade (given arming) reflects the *familiar gains from trade* that follow from canonical trade models based on comparative advantage.

4 Endogenous Security Policies

We now turn to the determination of non-cooperative equilibria in security policies and their dependence on trade regimes. Inspection of the objective functions under autarky (A) in (7) and under trade (T) in (10) reveals that the equilibrium production of the intermediate

[25] To be more precise, comparing $m_A^i = \mu(1, p_A^i)$ and $m_T^i = \mu(1, p_T^i)$ shows that, if $p_T^i < p_A^i = \alpha^i$ holds for both countries, then each country i specializes completely in the production of good i, and both countries are strictly better off under trade.

input, represented by the envelope function Z^i, is of central importance here. As noted earlier and established in Lemma A.1, given the countries' guns choices, Z^i is independent of the prevailing trade regime. Thus, the trade-regime dependency of arming incentives operates solely through a terms-of-trade channel.

To set the stage for this analysis, we make two additional observations that can be verified from (5).[26] First, the effect of a marginal increase in country i's gross endowment of capital $K_g^i = K^i + \phi^i K_0$ on Z^i is given by $Z_K^i = r^i/c^i$ and the effect of a marginal increase in its arming G^i (given K_g^i) on Z^i is $Z_{G^i}^i = -\psi^i/c^i$. Second, an increase in G^i also affects the rival's optimized production of the intermediate good Z^j through its influence the rival's gross capital endowment, $Z_K^j = r^j/c^j$. Bringing these observations together, while noting from (1) $\phi_{G^i}^j = -\phi_{G^i}^i$, shows:

$$\frac{dZ^i}{dG^i} = K_0 \phi_{G^i}^i Z_K^i + Z_{G^i}^i = \frac{1}{c^i}\left[r^i K_0 \phi_{G^i}^i - \psi^i\right] \tag{11a}$$

$$\frac{dZ^j}{dG^i} = K_0 \phi_{G^i}^j Z_K^j = -\frac{1}{c^j}[r^j K_0 \phi_{G^i}^i], \tag{11b}$$

given G^j, for $i \neq j = 1, 2$.

4.1 Autarky

As revealed by the payoff functions under autarky (7), since m_A^i is a constant, $\widehat{V}_A^i = \widehat{Z}^i$ holds. Thus, country i's arming choice influences V_A^i only through its effect on the maximized value of country i's intermediate input, Z^i. From (11a), each country i's first-order condition (FOC) for the choice of guns G^i, taking G^j as given, can be written as

$$\frac{1}{m_A^i}\frac{\partial V_A^i}{\partial G^i} = \frac{dZ^i}{dG^i} = \frac{1}{c^i}\left[r^i K_0 \phi_{G^i}^i - \psi^i\right] \leq 0, \quad i = 1, 2. \tag{12}$$

The first term in the brackets on the RHS of (12) reflects the marginal benefit of producing an additional gun for country i. Specifically, given G^j, an increase in G^i increases the share of the disputed resource K_0 that country i captures in the contest, thereby increasing the country's income and payoff. However, as shown in the second term, that additional gun comes at a cost in that capital and labor resources are diverted away from the production of consumption goods. Each country's optimal security policy balances this trade-off at the margin. Importantly, the negative influence of country i's security policy on country j's payoff through its effect on Z^j (shown in (11b)) does not directly enter this calculus.

Maintaining focus on the case in which the secure resource constraints on guns (i.e., $K^i - \psi_r^i G^i \geq 0$ and $L^i - \psi_w^i G^i \geq 0$) are not binding, equation (12) is an exact statement of country i's FOC under autarky. Furthermore, the conflict technology (1) implies interior

[26] Also see the proof of Lemma A.1 presented in Supplementary Appendix (B.1).

solutions with (12) holding as an equality. Since Z^i is concave in the country's own guns G^i and in the country's gross capital endowment K_g^i (see parts (c) and (e) of Lemma A.1) and K_g^i is concave in G^i through the conflict technology (1), we can show that V_A^i is strictly quasi-concave in G^i. This property ensures the existence of an interior (pure-strategy) equilibrium in security policies.

Let (G_A^1, G_A^2) be an equilibrium pair of guns. Some additional (but relatively mild) assumptions imply the equilibrium is unique:

Proposition 1 (*Equilibrium security policies under autarky.*) *An interior equilibrium in security policies exists under autarky: $G_A^i > 0$, for $i = 1, 2$. Furthermore, if labor and capital are sufficiently substitutable in the production of arms and/or the intermediate good, this equilibrium is unique.*

Observe from (12) that the equilibrium in security policies under autarky is independent of the elasticity of substitution in consumption. This independence follows since each country's problem under autarky is effectively one of maximizing income or, equivalently, the quantity of the intermediate good used in the production of traded goods. Matters differ, however, in the case of trade.

4.2 Free Trade

Security policies under trade, like those under autarky, affect payoffs through their impact on the output levels of the intermediate good. However, when trade is possible, these output changes also affect world prices as shown in (9). Using (9) and (10) and recalling that $\frac{\partial m_T^i / \partial p_T^i}{m_T^i / p_T^i} = -\gamma_j$, one can verify the following:

$$\widehat{V}_T^i = \widehat{Z}^i + \left(\frac{\partial m_T^i / \partial p_T^i}{m_T^i / p_T^i} \right) \widehat{p}_T^i = \widehat{Z}^i - \frac{\gamma_j}{\sigma} \left(\widehat{Z}^i - \widehat{Z}^j \right), \quad i \neq j = 1, 2. \tag{13}$$

The second term in the RHS of this expression shows the terms-of-trade effect.

Combining (11) with (13), the FOC for country i's arming choice becomes:

$$\frac{1}{m_T^i} \frac{\partial V_T^i}{\partial G^i} = \frac{1}{c^i} \left\{ \left[1 - \frac{\gamma_j}{\sigma} - \left(\frac{r^j / c^j Z^j}{r^i / c^i Z^i} \right) \frac{\gamma_j}{\sigma} \right] r^i K_0 \phi_{G^i}^i - \left[1 - \frac{\gamma_j}{\sigma} \right] \psi^i \right\} \leq 0, \tag{14}$$

for $i = 1, 2$ and $j \neq i$. Similar to the FOC under autarky (12), this FOC consists of two components: the marginal benefit and the marginal cost of an additional gun. However, the negative effect of an additional gun on country i's terms of trade modifies these two components differently as compared with autarky, due to the negative effect of an increase in i's guns on the rival's output Z^j. This differential effect suggests that whether trade is possible or not matters for the countries' incentives to arm.

Before exploring that influence, we characterize the equilibrium under trade. Henceforth, we assume that $\sigma > \gamma_j$ for $j = 1, 2$, such that the marginal cost of arming is strictly positive for both countries. This assumption, however, does not ensure that the marginal benefit, when evaluated at $G^i = 0$, is strictly positive. Digging a little deeper, let us define

$$\zeta^j \equiv \frac{r^i/c^i Z^i}{r^j/c^j Z^j + r^i/c^i Z^i} = \frac{c^j Z^j/r^j}{c^i Z^i/r^i + c^j Z^j/r^j}, \text{ for } i \neq j.$$

The parameter ζ^j reflects country j's relative size in terms of the countries' GDP, net of arming and measured in domestic units of the insecure resource. Then, the sign the marginal benefit for country i (i.e., the first term in (14)) is determined by the sign of $(\sigma - \gamma_j/\zeta^j)$. Hence, there exists a critical value of σ, $\overline{\sigma}^i \equiv \gamma_j/\zeta^j$ evaluated at $G^i = G^j = 0$ for each country $i \neq j = 1, 2$, such that country i's marginal benefit of arming when $G^i = 0$ is strictly positive (non-positive) if $\sigma > \overline{\sigma}^i$ ($\sigma \leq \overline{\sigma}^i$).

Since $\gamma_1 + \gamma_2 = \zeta^1 + \zeta^2 = 1$, there are two distinct possibilities to consider: (i) $\overline{\sigma}^1 = \overline{\sigma}^2 = 1$, which arises when the two countries have identical secure endowments;[27] and, (ii) $\overline{\sigma}^i > 1 > \overline{\sigma}^j$ for $i \neq j = 1, 2$, which arises in the presence of asymmetries. Thus, if $\sigma \leq 1$, the marginal benefit of arming must be non-positive for at least one country i (the relatively larger one) and possibly both; if $\sigma > 1$, the marginal benefit of arming must be positive for at least one country j (the relatively smaller one) and possibly both. Also observe that the maximum value of $\overline{\sigma}^i$ across i, $\overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$, is greater than or equal to 1.

Building on these ideas and using the FOC under trade (14), it is possible to verify that an equilibrium always exists. However, multiple pure-strategy equilibria and/or mixed-strategy equilibria are possible. Nonetheless, provided that the strength of the two countries' comparative advantage, represented by α^i, is sufficiently strong, a pure-strategy equilibrium that differs from the one under autarky exists under free trade.[28] The next proposition focuses on this case, letting (G_T^1, G_T^2) denote that equilibrium:

Proposition 2 (*Equilibrium security policies under free trade.*) *Suppose the conditions that ensure a unique equilibrium under autarky are satisfied and each country's comparative advantage (α^i) is sufficiently large. Then, there exists a pure-strategy equilibrium in security policies under free trade that is distinct from the equilibrium under autarky:* (*i*) $G_T^i = 0$

[27] Specifically, as discussed in Section 5.1, a symmetric equilibrium emerges with $Z^i = Z^j$, $\gamma_i = \gamma_j = \frac{1}{2}$ and $p_T^i = 1$ for $i = 1, 2$. Since equilibrium factor prices are identical across countries, we have additionally $\zeta^j = \frac{1}{2}$ in this benchmark case. Thus, $\sigma > 1$ ($\sigma \leq 1$) implies that the marginal benefit of arming is strictly positive (non-positive) at $G^i = 0$ for both i.

[28] The potential problem, as analyzed in Supplementary Appendix (B.1), is that p_T^i can vary only within the range $[1/\alpha^j, \alpha^i]$, giving rise to a possible discontinuity in country i's best-response function under trade such that it coincides with country i's best-response function under autarky for some G^j. Requiring that comparative advantage α^i be sufficiently large for both $i = 1, 2$ ensures the existence of a pure-strategy equilibrium in security policies that is distinct from the equilibrium under autarky. Nonetheless, even when this requirement is satisfied, we cannot rule out the possible existence of multiple equilibria in pure strategies.

for $i = 1, 2$ *if* $\sigma \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$ *and (ii)* G_T^i $(\neq G_A^i)$ > 0 *for* $i = 1, 2$ *if* $\sigma \in (\overline{\sigma}, \infty)$.

Observe the elasticity of substitution in consumption (σ) plays an essential role under free trade, as it determines the magnitude of the terms-of-trade effect. When the two goods are not close substitutes (i.e., $\sigma \leq 1$), then $\sigma \leq \overline{\sigma}^i$ for one country $i = 1$, 2, or both. Country i's payoff under trade in this case is highly dependent on country j's production, through relative world prices. Even though an increase in guns by country i, where initially $G^i = 0$ and given $G^j \geq 0$, generally implies a positive net marginal benefit of arming for given world prices (i.e., $r^i K_0 \phi^i_{G^i} - \psi^i > 0$), that additional gun by country i implies at the same time a worsening of its terms of trade. More precisely, it induces not only an increase in the supply of its exported good but also a decrease in the supply of its imported good. The terms-of-trade effect, then, tends to reduce the effective marginal benefit of an additional gun by more than it reduces the marginal cost. Indeed, when the two consumption goods are not sufficiently substitutable ($\sigma \leq 1$), the negative net terms-of-trade effect swamps the positive net marginal gain given world prices for at least one country, i. Then, given (1) with $\phi^i_{G^i} \in (0, \infty)$ at $G^i = 0$, country i chooses to produce no guns at all. If, in addition, $\sigma \leq \overline{\sigma}^j$, then $G_T^i = G_T^j = 0$. But, even if $\sigma > \overline{\sigma}^j$ while $\sigma \leq \overline{\sigma}^i$, our specification of the conflict technology (1) requiring that $f(0)$ be arbitrarily close to zero (even if positive) implies that country j's best response to $G^i = 0$ is to produce just an infinitesimal amount of guns. In this case, security costs are effectively equal to zero. Hence, we write in the case where $\sigma < 1 \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$, $G_T^1 = G_T^2 = 0$.[29]

When the two consumption goods are sufficiently substitutable (i.e., $\sigma > \overline{\sigma} \geq 1$), the negative effect through the terms of trade channel is not large enough to wipe out the positive net marginal benefit of arming given world prices when evaluated at $G^i = 0$ for either country i. The conditions specified in the proposition along with (1) ensure, in this case, the existence of an interior equilibrium in security policies under trade that differs from the equilibrium under autarky. At this internal equilibrium, the favorable effect of an increase in the rival country's arming G_T^j on country i's terms of trade is not sufficiently large to swamp the negative effect on country i payoff for given world prices. Put differently, the externality of each country j's arming remains negative in equilibrium: $dV_T^i/dG^j < 0$.[30]

[29] Observe $\sigma \leq 1$ is sufficient, but not necessary, for trade to effectively remove both countries' incentives to arm; the same result obtains when $\sigma > 1$, as long as $\sigma < \overline{\sigma}$.

[30] To confirm this claim, which also holds true when $\overline{\sigma}^i > \sigma > 1$ (such that country i chooses not to arm whereas country j chooses an infinitesimal quantity of arms), evaluate

$$\frac{\partial V_T^i/\partial G^j}{V^i} = \frac{dZ^i/dG^j}{Z^i} - \frac{\gamma_j}{\sigma}\left[\frac{dZ^i/dG^j}{Z^i} - \frac{dZ^j/dG^j}{Z^j}\right],$$

using the value of $\frac{dZ^i/dG^j}{Z^i}$ implied by country j's FOC under trade (14) for an interior solution, to find that a necessary and sufficient condition for $\partial V_T^i/\partial G^j < 0$ is that $\sigma > 1$.

5 Equilibrium Arming and the Relative Appeal of Trade

We are now in a position to compare the equilibrium payoffs under the two trade regimes, denoted by $V_A^{*i} \equiv V_A^i(G_A^i, G_A^j)$ in the case of autarky and $V_T^{*i} \equiv V_T^i(G_T^i, G_T^j)$ in the case of trade, for $i \neq j = 1, 2$. Combining Lemma 1 with the equilibrium analysis underlying Propositions 1 and 2, the next lemma establishes that a sufficient condition for trade to dominate autarky is simply that trade induce lower arming by both countries:

Lemma 2 (*Equilibrium payoffs under autarky vs. free trade.*) *Suppose that trade induces both countries to arm less as compared with autarky. Then, each one is strictly better off under free trade than under autarky (i.e., $V_T^{i*} > V_A^{i*}$ for $i = 1, 2$), and the difference in payoffs for both exceeds each country's standard gains from trade.*

The potential benefit for each country in moving to free trade can be decomposed into three parts. First, given both countries' arming choices, each country enjoys the standard gains from trade. As established in Lemma 1, these are non-negative for both countries and possibly positive for at least one if not both. Second, each country enjoys, given its own arming choice, a positive strategic effect as the opponent reduces its arming. Finally, each country's payoff rises, as it optimally adjusts its own arming choice in response to the new trade regime. Of course, in the presence of asymmetries in secure resources, one country could become less powerful under trade relative to its position under autarky. However, provided that both countries reduce their arming, each realizes lower security costs on top of the standard gains from trade.

Using Propositions 1 and 2, we now study the difference in equilibrium guns chosen by the two adversaries under the two trade regimes. Proposition 1 indicates the optimizing choice of guns for each country i under autarky is strictly positive. By contrast, Proposition 2 indicates that the equilibrium choice of guns for each country i equals zero when $\sigma \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$.[31] Thus, when the two goods are sufficiently complementary in consumption, equilibrium arming is lower under free trade and, by Lemma 2, welfare for each country is higher; each country enjoys not only the standard gains from trade, but also the elimination of security costs it induces.

When the two consumption goods are sufficiently substitutable (i.e., $\sigma > \overline{\sigma}$), each country's optimizing choice of guns is strictly positive under both autarky and free trade. But, a comparison of the FOCs under autarky (12) and trade (14) reveals the adverse terms-of-trade effect of a country's own arming reduces the marginal benefit of an additional gun relative to the analogous marginal benefit under autarky by more than it reduces the relative marginal cost. Thus, for any given G^j, country i's best response under free trade is strictly

[31] As noted above, arming by one country could be strictly positive, but would be infinitesimal.

less than its best response under autarky: $B_T^i(G^j) < B_A^i(G^j)$ for any $G^j \geq 0$, $i \neq j = 1, 2$.[32] This finding gives us a sufficient but not necessary condition for trade to induce lower equilibrium arming, when arming is strictly positive under both trade regimes—that is, if neither country's best-response functions exhibits strategic substitutability in the neighborhood of the autarkic equilibrium, then there exists a pure-strategy equilibrium under free trade where both countries choose lower arms and are strictly better off than under autarky.

Whether a country's best-response function is positively or negatively sloped in the neighborhood of the autarkic equilibrium generally depends on how an increase in the opponent's guns G^j influences country i's marginal benefit and marginal cost of arming (see equation (A.5) in the Appendix). As one can easily verify, an increase in G^j (given G^i) decreases K_g^i and thus, given L_g^i, reduces country i's equilibrium wage-rental ratio ω^i. Since country i's marginal cost of arming depends positively on ω^i (i.e., $\psi_\omega^i > 0$), the increase in G^j reduces country i's marginal cost and alone induces it to produce more guns. However, the effect of an increase in G^j on country i's marginal benefit of arming can be positive or negative. Specifically, equation (1) implies $\phi^i_{G^iG^j} \gtreqless 0$ as $B_A^i(G^j) \gtreqless G^j$. Thus, when $B_A^i(G^j) > G^j$ ($B_A^i(G^j) < G^j$), the implied positive (negative) effect of G^j on country i's marginal benefit of arming alone induces it to produce more (less) guns.

This analysis suggests that the sufficient condition for trade to lower equilibrium arming and enhance each country's payoff can be traced back to fundamentals that influence relative arming by the two countries under autarky. To examine this issue more carefully, we proceed in two steps. First, we focus in the next subsection on the following benchmark case, which we refer to as *complete symmetry*: in addition to having identical preferences (defined symmetrically over the two consumption goods) and identical technologies in the production of G^i and Z^i, the two contending countries have identical endowments of secure capital and labor, $K^i = K$ and $L^i = L$ for $i = 1, 2$. Then, in the following subsection, we consider asymmetric distributions. The appeal of this approach over considering, for example, differences in the countries' technologies for producing guns stems from the fact that relative endowments are observable and contain information regarding differences in the degree of resource security across countries. Throughout, we assume sufficient substitutability between consumption goods (i.e., $\sigma > \overline{\sigma}$) such that arming is strictly positive

[32] Using (13) to find

$$\frac{\partial V_T^i/\partial G^i}{V_T^i} = \left(1 - \frac{\gamma_j}{\sigma}\right)\frac{dZ^i/dG^i}{Z^i} + \left(\frac{\gamma_j}{\sigma}\right)\frac{dZ^j/dG^i}{Z^j},$$

and recalling from (11b) that $dZ^j/dG^i < 0$, it is possible confirm our assumption $1 - \gamma_j/\sigma > 0$ that ensures a positive marginal cost of arming is both sufficient and necessary to rule out the possibility that, given G^j, $dZ^i/dG^i < 0$ at an interior optimum for country $i \neq j = 1, 2$ under trade. Thus, the finding that trade openness lowers each country i's arming incentive (given G^j) cannot be attributed to the presence of immizerising growth (Bhagwati, 1959).

under trade as well as under autarky.

5.1 Complete Symmetry

Suppose that the two countries arm identically: $G^1 = G^2$. Regardless of the trade regime in place, this assumption under the conditions of complete symmetry generally implies from (1) that $\phi^i = \frac{1}{2}$ for $i = 1, 2$, which in turn implies that each country holds, upon dividing the contested resource K_0, an identical amount of capital to employ in the production of the intermediate input: $K_Z^i = K_Z$ for $i = 1, 2$. Then, (4) and (5) imply identical relative factor prices and production of the intermediate good: $\omega^i = \omega$ and $Z^i = Z$ for $i = 1, 2$. The unit costs of producing Z and G, measured in units of capital (respectively c^i/r^i and ψ^i/r^i), are also identical across countries.

Under autarky, these simplifications mean that the marginal benefit and the marginal cost of arming are identical across countries. As such, from the FOC under autarky (12), identical arming across countries is, in fact, consistent with optimizing behavior by both and thus represents a possible equilibrium. From Proposition 1, we know further that, provided labor and capital are sufficiently substitutable in the production of the intermediate good and guns, the equilibrium is unique, and arming is strictly positive regardless of the elasticity of substitution in consumption: $G_A^i = G_A > 0$ for $i = 1, 2$ and all $\sigma > 0$.

Under free trade, the simplifications that follow from complete symmetry imply $p_T^i = 1$ for $i = 1, 2$, and thus $\gamma_1 = \gamma_2 = \frac{1}{2}$ when $G^1 = G^2$. Accordingly, from the relevant FOC (14) and our maintained assumptions that comparative advantage for each country is sufficiently strong and the two final goods are sufficiently substitutable (i.e., $\sigma > \bar{\sigma}^i = 1$), Proposition 2 implies the equilibrium in security policies under free trade is also unique and symmetric, with strictly positive arming: $G_T^i = G_T > 0$ for $i = 1, 2$.[33] More importantly for our purposes here, since the two countries arm identically under autarky, both countries' best-response functions necessarily exhibit strategic complementarity in the neighborhood of the autarkic equilibrium, as illustrated in Fig. 1(a). Hence, starting at the autarkic equilibrium (point A in the figure where $G_A^i = G_A > 0$ for $i = 1, 2$), a shift to free trade induces both countries' best-response functions to shift inward to intersect at a new equilibrium with less arming by both countries (point T in the figure where $0 < G_T^i = G_T < G_A$ for $i = 1, 2$). The effect of trade to induce each country to produce fewer arms than under autarky, without influencing the distribution of the contested resource, implies that both countries enjoy equally lower security costs under trade. Thus, as implied by Lemma 2, a shift to free trade is welfare-improving for both countries, and the gains they realize are strictly greater than those predicted by the traditional paradigm that abstracts from conflict all together.[34]

[33] When $\sigma < 1$, we have $G_T = 0$.

[34] Proposition B.1 presented in Supplementary Appendix B.1 shows G_T is increasing in the elasticity of substitution in consumption for $\sigma > 1$ $(= \overline{\sigma})$, approaching G_A (which is independent of σ) as $\sigma \to \infty$.

5.2 Resource Asymmetries

While our results above illustrate how the pacifying effect of trade openness can follow intuitively from a formal game-theoretic analysis, the assumption of complete symmetry is not innocuous. In this subsection, we explore specifically the possible importance of asymmetries in the initial distribution of secure resources. We start by asking whether there exist asymmetric distributions of secure resources that similarly imply a symmetric equilibrium in security policies under autarky and thus satisfy the sufficient condition for trade to induce lower arming. The answer is yes.

In particular, we can establish that there exists a set of initial distributions of secure resources (aside from the symmetric distribution) that similarly imply a symmetric equilibrium in security policies under autarky. To this end, recall that $k_Z^i \equiv K_Z^i/L_Z^i$ denotes country i' residual capital-labor ratio. Equation (4) shows that $k_Z^i = c_r^i/c_w^i$ determines the equilibrium wage-rental ratio ω_A^i, and these ratios are identical across the two countries under the conditions of complete symmetry in the autarkic equilibrium: $k_{ZA}^i = k_{ZA}$ and $\omega_A^i = \omega_A$ for $i = 1, 2$. Now, fix each country's guns production at G_A and reallocate both labor and secure capital from country 2 to country 1 such that $dK^i = k_{ZA} dL^i$ for $i = 1, 2$, so as to leave their residual capital-labor ratios unchanged at k_{ZA}. (For future reference, denote the set of the resulting distributions of secure resources by $\mathcal{S}_0$.) But, since k_{ZA} does not change by assumption, the equilibrium value ω_A also remains unchanged. Thus, by the FOC under autarky (12), neither country has an incentive to change its arming with the redistribution of labor and capital. As such, for asymmetric distributions of secure resources in $\mathcal{S}_0$, equilibrium arming under autarky remains the same as when secure resources are evenly distributed across countries. Since arming also remains identical across countries, their best-response functions continue to exhibit strategic complementarity in the neighborhood of the autarkic equilibrium. Therefore, for distributions of secure endowments within $\mathcal{S}_0$ (including but not limited to the case of complete symmetry), there exists a pure-strategy equilibrium under free trade with less arming by each country than under autarky (i.e., $G_T^i < G_A$ for $i = 1, 2$) and thus by Lemma 2 greater payoffs for both countries.[35]

Now, consider a slightly different experiment. Starting from any distribution of secure resources in $\mathcal{S}_0$, transfer one unit of labor from country 2 to country 1. One can confirm this sort of transfer for given arming choices increases the relative wage and thus the marginal

Thus, consistent with Hirshleifer's (1991) argument, the savings in security costs afforded by trade equal zero when the two goods are perfect substitutes and increase as σ falls approaching 1 (or equivalently as the two economies become increasingly integrated).

[35] Note that, successive transfers of secure labor and capital resources from country 2 to country 1 within $\mathcal{S}_0$ eventually imply $\sigma - \gamma_2/\zeta^2 < 0$ and thus drive arming by both countries under free trade (effectively) to zero. More generally, uneven distributions of secure resources in $\mathcal{S}_0$ imply that the two countries produce different quantities of Z^i, and such differences cause their FOC's under trade (14) to differ in equilibrium, such that $G_T^1 \neq G_T^2$ even though $G_A^1 = G_A^2$.

cost of arming in the donor country (2) and has the opposite effects in the recipient country (1).[36] Thus, by the FOC under autarky (12) along with the strict quasi-concavity of payoffs under autarky demonstrated in the proof to Proposition 1, this transfer of labor induces both countries' best-response functions under autarky to rotate clockwise, with country 1 producing more guns and country 2 producing less: $G_A^1 > G_A^2$. The logic spelled out above, in turn, implies that country 1's best-response function continues to be positively sloped in the neighborhood of the autarkic equilibrium. Furthermore, for small transfers of labor from country 2 to country 1, country 2's best-response function also remains positively sloped. By continuity, then, both countries' best-response functions exhibit strategic complementarity for distributions adjacent to $\mathcal{S}_0$, and the result that $G_A^i > G_T^i$ for $i = 1, 2$ remains intact. As one can verify, transfers of secure capital this time from country 1 to country 2, again starting from any distribution of secure resources in $\mathcal{S}_0$, generate similar effects. Hence, provided that the distribution of initial resource endowments across the two countries imply sufficiently similar residual capital-labor ratios, a shift from autarky to trade will lower equilibrium arming and thus, by Lemma 2, will bring positive gains to both countries that exceed the gains predicted by mainstream models that ignore conflict.

Nevertheless, we cannot rule out the possibility that a shift from autarky to free trade implies greater arming by one country. A necessary condition, suggested by our discussion above, is that the mix of secure labor and capital resources held initially by the two countries be sufficiently uneven to generate large differences in the countries' arming choices under autarky that make one country's best-response function negatively related to the opponent's arming in the autarkic equilibrium.[37] But, even in this case, it is possible that a move to trade induces lower equilibrium arming by both countries. A more extreme asymmetry in secure resource endowments across countries is required for trade to induce one country to become more aggressive, as illustrated in Fig. 1(b), which assumes that country 2 is the country having the higher capital-to-labor ratio and thus arms less under autarky than its rival.[38] Starting from the autarkic equilibrium depicted by point A in the figure, a shift to free trade causes each country's best-response function to shift inward towards the 45° line, resulting in a new intersection at point T, with decreased arming by country 1 but increased arming by country 2.

What are the welfare implications of trade when trade induces one country (2) to expand

[36] See Lemma A.1(b) presented in the Appendix.

[37] Observe from equation (A.5) in the Appendix, the conflict technology (1) implies it is not possible for both countries' best-response functions to be negatively sloped in the neighborhood of the autarkic equilibrium. Still, there exist extremely large transfers of labor that could induce both countries to produce less guns relative to the benchmark (symmetric) equilibrium.

[38] Note the scale for G^1 in the figure is more concentrated such that the slope of the "45°" line drawn is greater than 45°. Numerical simulations based on a particular parameterization of the model confirm that an increase in arming by one country is possible only under extremely uneven secure distributions of the two primary resources. Details are available from the authors upon request.

its arming? Because country 1 arms less heavily under trade than under autarky, country 2 enjoys a positive strategic welfare effect as well as the standard gains with a move from autarky to trade. By contrast, due to the adverse strategic effect of country 2's arming, it is possible for trade to reduce country 1's payoff. Such a preference ranking is more likely to hold when country 1's strength of comparative advantage (α^1) and thus the standard gains it realizes from trade are relatively small. Either way, the possibility that trade could induce greater arming by one country under some circumstances illustrates one potential (although perhaps remote) limit to the classical liberal view.

5.3 The Effects of Trade Costs

Given our focus above on free trade, one might naturally wonder how trade costs matter. In this subsection we extend the analysis to study the importance of geographic trade costs and import tariffs for arming and payoffs. Henceforth, suppose that, under trade, each country i's demand for good j is entirely satisfied by imports: $M^i = D^i_j$ for $j \neq i = 1, 2$. Let τ^i and t^i respectively denote one plus an iceberg type transportation cost and ad valorem tariff rate on its imports, and define p^i_T and q^i_T as the corresponding internal and external relative prices of the same product. Arbitrage implies that, in a trade equilibrium with positive trade flows, $p^i_T = \tau^i t^i q^i_T$ holds. As shown in Supplementary Appendix B.2, the percentage change in i's payoff V^i_T under trade is

$$\begin{aligned} \widehat{V}^i_T &= \left(1-\rho^i\right)\widehat{Z}^i + \rho^i \widehat{Z}^j - \rho^i\left[\varepsilon^j \widehat{\tau}^i + \left(\varepsilon^j - 1\right)\widehat{\tau}^j + \eta^j \widehat{t}^j\right] \\ &\quad + \left(\gamma^i_j / t^i \Delta\right)\eta^i\left[1-\left(t^i-1\right)\left(\varepsilon^j-1\right)\right]\widehat{t}^i, \end{aligned} \tag{15}$$

for $j \neq i = 1, 2$, where

$$\begin{aligned} \varepsilon^i &\equiv -\frac{\partial M^i / \partial p^i_T}{M^i / p^i_T} = 1 + (\sigma - 1)\frac{t^i \gamma^i_i}{t^i \gamma^i_i + \gamma^i_j} \\ \eta^i &\equiv -\left.\frac{\partial M^i / \partial p^i_T}{M^i / p^i_T}\right|_{dU^i=0} = \varepsilon^i - \frac{\gamma^i_j}{t^i \gamma^i_i + \gamma^i_j} = \sigma\left(\frac{t^i \gamma^i_i}{t^i \gamma^i_i + \gamma^i_j}\right) > 0 \\ \Delta &\equiv \varepsilon^i + \varepsilon^j - 1 \\ \rho^i &\equiv \left[\frac{\left(t^i - 1\right)\varepsilon^i + 1}{t^i}\right]\left(\frac{\gamma^i_j}{\Delta}\right). \end{aligned}$$

Note that ε^i (resp., η^i) is the absolute value of country i's Marshallian (compensated) price elasticity of import demand. Also note that $\text{sign}\{\varepsilon^i - 1\} = \text{sign}\{\sigma - 1\}$. For specificity and clarity, hereafter we focus on the case where $\sigma > 1$, which implies $\Delta > 0$ and $\rho^i \in (0, 1)$.

With the help of equation (15) and our analysis of security policies under autarky we now arrive at

Lemma 3 (*Arming incentives under trade.*) *For any given guns $G^j > 0$ and trade cost* *levels that induce positive trade flows, we have $B^i_T(G^j) < B^i_A(G^j)$ for $j \neq i = 1, 2$.*

This lemma generalizes our analysis in the beginning of Section 5, showing that a move from autarky to trade, whether free or distorted with tariff and/or non-tariff barriers, reduces arming incentives for each country given the rival's arming. What's more, the result remains valid even if tariffs are chosen non-cooperatively and simultaneously with security policies.[39] Building on this lemma, one can establish the following:

Proposition 3 (*Equilibrium arming.*) *Provided an equilibrium with strictly positive trade flows exists, we always have:* (a) $G^i_T < G^i_A$ *for at least one country* $i \in \{1, 2\}$*; and* (b) $G^1_T + G^2_T < G^1_A + G^2_A$.

Thus, as in the case of free trade, trade distorted with tariff and non-tariff barriers induces one adversary (and possibly both) to reduce its arming below the autarkic level. Still, it is possible for one adversary to produce more guns under trade than under autarky.[40] Nevertheless, regardless of how it affects each country's security policy, trade always brings about a reduction in aggregate arming.

To shed light on the above comparison and also to examine the dependence of arming and welfare on trade costs, we proceed with the help of numerical methods.[41] Let us start with geographic trade costs (τ^i). We find that, under most circumstances, globalization (i.e., $\tau^i \downarrow$ for $i = 1$, 2 or both) reduces both countries' equilibrium arming. To be sure, consistent with our analysis in Section 5.2, there do exist sufficiently uneven international distributions of secure resources to imply that the smaller country j's arming rises τ^i falls (i.e., $dG^j_T/d\tau^i < 0$). However, exhaustive numerical analysis (assuming $\alpha^i = \infty$) confirms that, under all circumstances, aggregate arming falls and each country's payoff rises.

The analysis of tariffs (t^i) is, as one would imagine, more complex and thus more difficult to characterize in a succinct way. Specifically, the effect of an increase in t^i on arming depends not only on the distribution of secure resources but also on t^j and the initial level of t^i, reflected in non-monotonicities in both equilibrium arming and payoffs. Nonetheless, we obtain some important results when comparing various equilibrium outcomes, including autarky (A), free trade (F) and "generalized war" (W), where (in the last case) both

[39] As in standard analyses of trade wars that abstract from resource disputes and the associated resource costs, autarky is always a possible equilibrium when tariff policies are chosen non-cooperatively; however, as suggested by the previous literature, an interior equilibrium in tariff policies normally exists as well even when the countries differ in size (Syropoulos, 2002).

[40] As before, a necessary condition for this possibility is that the aggressive country's arming under autarky is a strategic substitute for its rival's arming. In turn, this condition requires the international distribution of secure resource endowments to be sufficiently asymmetric.

[41] Due to space limitations, we highlight only some results here. The interested reader is referred to Supplementary Appendix B.2 for details and additional results.

countries simultaneously and non-cooperatively choose their security and trade policies.[42] Consistent with our priors based on the analysis above and with the idea that trade wars have a prisoner-dilemma feature, we find that under most circumstances, $G^i_A > G^i_W > G^i_F$ and $V^i_A < V^i_W < V^i_F$. However, when one country (i) is extremely larger than its rival (j), a different ranking of arming emerges for both countries. The smaller country (j) arms more heavily under free trade than under autarky, and it arms by even more under generalized war: $G^j_W > G^j_F > G^j_A$. Nonetheless, the ranking of its payoffs is the same as when the countries are more equally sized. While the much larger country (i) continues to arm by less under free trade than under autarky, its arming under generalized war is the lowest ($G^i_A > G^i_F > G^i_W$), suggesting that its extremely larger size renders its trade policy much more effective in influencing its terms of trade. Furthermore, similar to Syropoulos' (2002) finding in a setting with secure property rights, we also find that this country prefers generalized war over the other outcomes: $V^i_A < V^i_F < V^i_W$. What is perhaps surprising is that this extremely large country prefers all these outcomes to those that would obtain if there were no resource insecurity and thus no arming at all.

6 Trading with Friends

While the analysis above provides analytical support to the notion that trade can be pacifying in international relations, our focus has been on two economically interdependent countries. In this section, we explore how the nature of the trading relationship between the contending nations can matter, showing a possible limit to the optimism of the classical liberal view. In particular, in contrast to our setting above where differences in technology to produce consumption goods render trade between contending countries mutually advantageous, we now consider a setting where the structure of technology is such that the two contending countries do not trade with each other, but instead trade with a third, non-adversarial country. Extending the two-country, two-good model analyzed above to a three-country, two-good model, we show that a shift to free trade can induce greater arming by both contending countries, even under the conditions of complete symmetry.[43]

Suppose the two adversarial countries ($i = 1, 2$) are identical in all respects, including the comparative advantage they enjoy in producing good 1. Specifically, assume $a^1_1 = a^2_1 = 1$ and $a^1_2 = a^2_2 \equiv a_2 > 1$, implying the relative price of good 2 under autarky in both countries ($p^i_A = p_A$ for $i = 1, 2$) satisfies $p_A = a_2$. Although only two goods are produced as before ($j = 1, 2$), there is now a third country ($i = 3$). Consumer preferences are identical worldwide, but country 3 is not involved in the contest over K_0. Moreover, country 3 has a comparative advantage in producing good 2: $a^3_1 > a^3_2 = 1$, which implies that its relative

[42] The last case has been extensively considered in the literature on trade policies with secure property (e.g., Johnson, 1953-54; Kennan and Riezman, 1988; and, Syropoulos, 2002).

[43] Applying our earlier logic, the results to follow are robust to non-prohibitive trade costs.

price of good 1 under autarky, $p_A^3 = a_1^3$, satisfies the inequality $1/p_A^3 < p_A$. Not surprisingly, the introduction of a third, friendly country has no relevance for equilibrium arming and payoffs under autarky.

When trade is possible, the assumed production structure implies countries 1 and 2 export good 1 in return for imports of good 2 from country 3. In a trade equilibrium, then, all three countries face the same prices for goods 1 and 2, respectively denoted by p_1 and p_2, and thus the same relative price $p_T \equiv p_2/p_1$ for good 2, and this price balances world trade.[44] With each country specializing in the good in which it has a comparative advantage, balanced trade requires $p_2(D_2^1 + D_2^2) = p_1 D_1^3$. Since the countries have identical and homothetic preferences and face identical world prices, their expenditures shares are identical. Furthermore, specialization in production implies $Y^i = p_1 X_1^i = p_1 Z^i$ for $i = 1, 2$, and $Y^3 = p_2 X_2^3 = p_2 Z^3$. It then follows that $D_2^i = \gamma_2 \frac{p_1 Z^i}{p_2}$ $(i = 1, 2)$ and $D_1^3 = \gamma_1 \frac{p_2 Z^3}{p_1}$. The world market-clearing condition, then, implies $p_T(\equiv \frac{p_2}{p_1}) = \gamma_2(Z^1 + Z^2)/\gamma_1 Z^3$, where $\gamma_2 = (p_T)^{1-\sigma}/[1+(p_T)^{1-\sigma}]$ and $\gamma_1 = 1-\gamma_2$. Now differentiate this expression logarithmically, while keeping the intermediate output of country 3 (Z^3) fixed in the background, and rearrange to find: $\widehat{p}_T = \frac{1}{\sigma}(\nu^1 \widehat{Z}^1 + \nu^2 \widehat{Z}^2)$, where $\nu^i \equiv Z^i/(Z^1 + Z^2)$ for $i = 1, 2$ and $\sigma > 0$.

As in the two-country case, a country's security policy under trade affects the relative price of its imported good through its impact on Z^i for $i = 1, 2$, that satisfies (4) and (5). This effect, in turn, makes the incentive to arm trade-regime dependent.[45] Furthermore, as before, an increase in arming by country i reduces the opponent's production of the intermediate good given G^j: $dZ^j/dG^i < 0$ for $i \neq j = 1, 2$, as shown in (11b). The key difference here is that the decrease in Z^j now improves country i's terms of trade. To see how this difference matters in determining arming incentives, note first that each adversary's payoff function under free trade V_T^i can be written as before and as shown in (10), but now where $m_T^i = \mu(1, p_T)$ with $\frac{\partial m_T^i/\partial p_T}{m_T^i/p_T} = -\gamma_2$ for $i = 1, 2$. Then differentiate V_T^i to obtain

$$\widehat{V}_T^i = \widehat{Z}^i + \left(\frac{\partial m_T^i/\partial p_T}{m_T^i/p_T}\right)\widehat{p}_T = \widehat{Z}^i - \frac{\gamma_2}{\sigma}\left(\nu^i \widehat{Z}^i + \nu^j \widehat{Z}^j\right). \tag{16}$$

Using (11) in (16), country i's FOC under free trade can be written as:

$$\frac{1}{m_T^i}\frac{\partial V_T^i}{\partial G^i} = \frac{1}{c^i}\left[\left(1 - \frac{\nu^i\gamma_2}{\sigma} + \frac{\nu^j\gamma_2}{\sigma}\left[\frac{r^j/c^j Z^j}{r^i/c^i Z^i}\right]\right) r^i K_0 \phi_{G^i}^i - \left(1 - \frac{\nu^i\gamma_2}{\sigma}\right)\psi^i\right] \leq 0, \tag{17}$$

[44] As before, we focus on the case where the world price of country 1 and 2's imported good in terms of their exported good satisfies $p_T \in (1/p_A^3, p_A)$ to abstract from the potential complications associated with discontinuities in the best-response functions. However, we briefly discuss these issues in connection with the detailed proof of Proposition 4(b), presented in Supplementary Appendix (B.1).

[45] Note, if the two adversarial countries were so small that their production of the intermediate input had no influence on p_T, their security policies would not generate a terms-of-trade effect and therefore would not be trade-regime dependent. The sharp contrast of this result with that of Garfinkel et al. (2015) stems from the presence of a factor-price channel that is absent in the present analysis.

for $i \neq j = 1, 2$. The first term inside the outer square brackets reflects the marginal benefit of arming, whereas the second term reflects the marginal cost. Assuming $\sigma > \nu^i \gamma_2$ ensures that both terms are positive for each adversary. Then, using (17) and the FOC under autarky (12) for each country i, one can show the following:[46]

Proposition 4 (*Equilibrium arming and payoffs under free trade with a third, friendly country.*) *Suppose that two identical adversarial countries potentially compete in the same market for exports to a third, friendly country. Then, a shift from autarky to free trade* (a) *induces each adversary to arm more heavily and* (b) *can be welfare reducing for both.*

The intuition for part (a) is that, with trade, each country has an interest in producing more guns at the margin not only to appropriate more K_0 and thus produce more Z, but also to reduce its rival's output, thereby improving its terms of trade.[47] Additionally, as one can verify, an increase in the substitutability of consumption goods (σ) reduces the magnitude of this terms-of-trade effect.[48] But, as established in part (b), with intensified conflict between the two contending countries, free trade brings higher security costs, and these higher security costs can swamp the gains from trade. In the proof, we establish this possibility based on sufficiently weak comparative advantage that makes the gains from trade sufficiently small.[49]

7 Empirical Evidence: The Effects of Trade between Enemies versus Friends on Military Spending

Our theory generates testable predictions regarding the impact of trade on military spending. Specifically, consistent with the classical liberal view, the main analysis in Section 5 implies that trade between rivals should have a negative impact on their military spending; by contrast, the analysis of Section 6 suggests that trade between between countries viewed as friends should have a positive effects on their military spending, provided they have

[46] As noted earlier, we assume here that $p_T \equiv p_2/p_1 \in (1/p_A^3, p_A)$.

[47] In its quest for raw resources with an aim to match Great Britain's access and thus be better able to compete with Great Britain in the export of manufactures to third-countries, Germany invaded parts of Eastern Europe at the outset of WWII. Eventually shifting its efforts westward, Germany had hoped that it could negotiate some sort of peaceful settlement with Great Britain. But, of course, no such settlement was reached. Our analysis suggests that, insofar as Germany and Great Britain did not trade with each other, each side had an interest to fight, even if costly, for terms-of-trade (among other) reasons.

[48] In the limiting case where $\sigma = \infty$, this effect vanishes and arming incentives for both adversaries are identical across the two trade regimes.

[49] Furthermore, we expect that admitting the possibility of trade in arms between the third friendly country and each of the two adversaries would not change our results qualitatively and, in fact, could amplify the positive effect of trade on arming incentives, implying even greater security costs and a larger likelihood of negative welfare consequences.

rivals. To test these predictions, we use the following econometric model:

$$\begin{aligned} \mathit{MLTRY\_SPEND}_{i,t} &= \beta_0 + \beta_1 \mathit{TRADE\_RIVALS}_{i,t} + \beta_2 \mathit{TRADE\_FRIENDS}_{i,t} \\ &\quad + \beta_3 \mathit{TRADE\_NR}_{i,t} + \mathbf{CONTROLS_{i,t}} \kappa + \xi_t + \xi_i + \epsilon_{i,t}, \end{aligned} \tag{18}$$

where $\mathit{MLTRY\_SPEND}_{i,t}$ is the logarithm of military expenditure in country i at time t, taken from the Stockholm International Peace Research Institute (SIPRI). Data for the trade variables on the RHS of the specification come from Baier et al. (2016) and are based on total manufacturing trade data from UNCTAD's COMTRADE.[50] The two key covariates of interest are $\mathit{TRADE\_RIVALS}_{i,t}$ and $\mathit{TRADE\_FRIENDS}_{i,t}$, reflecting the trade activity of countries that have rivals, classified as such using data on "strategic rivalries" between country-pairs from Thompson (2001).[51] $\mathit{TRADE\_RIVALS}_{i,t}$, in particular, is defined as the logarithm of total exports of country i to its rivals at time t. Our theory predicts $\beta_1 < 0$. $\mathit{TRADE\_FRIENDS}_{i,t}$ is defined as the logarithm of total exports for country i (which has rivals) to its friends. According to our theory, we expect $\beta_2 > 0$.

In addition to the theoretically motivated covariates in (18), we also control for other potential determinants of military spending. First, since only a subset of the countries in our sample have rivals, we add $\mathit{TRADE\_NR}_{i,t}$, defined as the logarithm of total exports of country i having no rivals. The vector $\mathbf{CONTROLS_{i,t}}$ includes proxies for country size, such as the logarithm of Gross Domestic Product ($\mathit{GDP}_{i,t}$) and the logarithm of i's population ($\mathit{PPLN}_{i,t}$). We also control for the stability of a country's government ($\mathit{INSTITUTIONS}_{i,t}$), for whether a country has rivals or not ($\mathit{RIVALS}_{i,t}$), and for the extent to which the country is involved in geopolitical conflict ($\mathit{HOSTILITY}_{i,t}$). Data on the additional covariates come from the dynamic gravity data set of the U.S. International Trade Commission, and we refer the reader to Gurevich and Herman (2018) for further details on this database. Combining these data resulted in an unbalanced panel dataset covering 64 countries over the period 1986–1997.[52] Finally, we include year fixed effects (ξ_t) to capture any global trends, and country fixed effects (ξ_i) to account for any observable and unobservable country characteristics that may affect military spending. To estimate equation (18), we use the OLS estimator with robust standard errors.

The main estimates appear in Table 1. To establish the robustness of our findings and to better understand the channels through which trade affects military expenditure, we start with simple correlations to which we build on gradually. Column (1) of Table 1 reports

[50] We obtain similar results with IMFs Direction of Trade Statistics database.

[51] More precisely, Thompson's (2001) indicator records whether or not there exists a threat of war between two countries based on observed diplomatic tensions; as such, two countries need not be involved in a war to be classified as "strategic rivals."

[52] The starting date of our sample was determined by the trade dataset of Baier et al. (2016), while the ending date was set by the strategic rivals dataset from Thompson (2001). The countries included are listed Table B.1 of Supplementary Appendix B.

the results when we include only the three trade covariates. The estimates of the effects of $TRADE\_RIVALS_{i,t}$ and $TRADE\_FRIENDS_{i,t}$ are precisely as predicted by our theory. Specifically, the negative estimate of β_1 suggests an inverse relationship between military spending and trade with rivals, while the positive estimate of β_2 supports the prediction of a positive relationship between military spending and trade with friends. Finally, the large, positive, and statistically significant estimate on $TRADE\_NR_{i,t}$ captures a direct relationship between trade and military spending for countries without rivals. While our theory does not speak directly to this result, we demonstrate below that it is robust and we find it interesting, especially because it contrasts with the classical liberal view.

The estimates from Column (2) of Table 1, which introduces the additional control variables, are mostly intuitive. The large, positive, and significant estimate on $GDP_{i,t}$ suggests that larger countries devote more resources to military spending. The negative estimate on $PPLN_{i,t}$ means that, all else equal, countries with more people invest less in military spending.[53] The negative estimate on $INSTITUTIONS_{i,t}$ shows, all else the same, countries with stronger institutions have lower military expenditures. We also find intuitive the positive estimate on $HOSTILITY_{i,t}$, which suggests that countries with more hostile environments devote more resources to their militaries. The estimate on $RIVALS_{i,t}$ is not statistically significant. Finally, we note that, while smaller in magnitude, the estimates on the key trade covariates retain their signs and remain sizable and statistically significant.

We finish the empirical analysis by addressing the important concern, raised by Acemoglu and Yared (2010), of possible endogeneity of the trade variables in our econometric model. To that end, we proceed in three steps. First, in column (3) of Table 1 we introduce year and country fixed effects. The inclusion of these variables, especially the country fixed effects, should absorb most of the unmodeled correlation between the endogenous trade variable and the error term. The estimates in column (3) reveal that, while the proxies for size are no longer statistically significant, the estimates of the effects of the key trade covariates remain sizable and statistically significant with signs as predicted by our theory. Second, to address simultaneity concerns, in column (4) of Table 1, we use lagged values for the three trade covariates. The results remain qualitatively identical and quantitatively similar. Finally, in column (5) of Table 1, we employ an instrumental variable estimator, where we instrument for each of the three potentially endogenous trade variables with their lags and with instruments that are constructed based on gravity regressions following the methods of Frankel and Romer (1999) and Feyrer (2009).[54] Two findings stand out from

[53] We note that if we do not include $GDP_{i,t}$, then the estimate on $PPLN_{i,t}$ becomes positive and significant capturing the impact of size on military spending.

[54] Specifically, similar to Frankel and Romer (1999), we use as instruments constructed export values that are based on predicted bilateral trade flows from gravity regressions. However, following the criticism and recommendations of Feyrer (2009), we do not use exporter and importer fixed effects. Instead, we use the

the estimates in column (5). First, by rejecting the null hypothesis with an UnderId statistic of $\chi^2 = 30.998, (p - val = 0.000)$, our instruments pass the under-identification test of whether the excluded instruments are correlated with the endogenous regressors. This result is reinforced by the "weak identification" (WeakId) Wald F statistic of 8.038 (Kleibergen and Paap, 2006).[55] In addition, our instruments also pass the over-identification test with $\chi^2 = 3.964, (p - val = 0.265)$. Second, once again and most important, the estimates of the coefficients on the two key trade variables in column (5) remain statistically significant and have signs as predicted.

8 Concluding Remarks

This paper develops a Ricardian model of trade, augmented by conflict between two countries over a productive resource. It features a terms-of-trade channel that renders arming choices trade-regime dependent. Specifically, a country's arming under trade has an additional effect on its own payoff, by influencing the adversary's production and thus world prices. Exactly how trade influences arming incentives depends on the structure of comparative advantage and trade costs.

When the two countries in conflict also trade with each other, the impact of a country's arming on its terms of trade is negative. Provided these countries are sufficiently symmetric, not only in terms of technologies and preferences, but also in terms of the mix of their secure resource endowments, equilibrium arming by both is lower and their payoffs higher under trade than under autarky. These results, which are robust to the presence of trade costs, provide theoretical support to the long-standing classical liberal hypothesis that increased trade openness can ameliorate conflict and thus amplify the gains from trade. With sufficiently extreme differences in the distribution of the primary resources, a shift to trade could induce one country to arm more heavily and to such an extent so as to imply that autarky is preferable over trade to the other country.[56] Nevertheless, in an equilibrium that involves positive trade flows, aggregate resources allocated to dispute the contested resource are lower than in an equilibrium with no trade at all.

When the structure of comparative advantage is such that the two adversaries do not trade with each other, but instead trade with a third, friendly country and they compete in

logarithms of the population of the exporter and of the importer countries. Thus, our underlying gravity regressions include the standard exogenous bilateral gravity variables (e.g., distance, contiguity, language, etc. as well as the logarithms of exporter and importer populations).

[55]The Kleibergen-Paap Wald test is appropriate when the standard error i.i.d. assumption is not met and the usual Cragg-Donald Wald statistic (Cragg and Donald,1993), along with the corresponding critical values proposed by Stock and Yogo (2005), are no longer valid. This is true in our case, where the standard errors are either robust or boostrapped.

[56]In ongoing research, we have extended the the analysis to capture the presence of a non-tradable goods sector, finding that, even in the case of complete symmetry, different factor intensities across the tradable and non-tradable sectors can cause the classical liberal view to fail.

the same export market, the terms-of-trade effect of security policies is positive. As such, a shift from autarky to trade unambiguously intensifies international conflict, possibly with negative net welfare consequences.

Consistent with the model's predictions, our empirical analysis provides reduced-form evidence that the effects of trade on a country's military spending depend qualitatively on whether trade is with a rival or with a friend. Thus, our findings complement the more structural evidence presented by Martin et al. (2008), that increased opportunities for multilateral trade can aggravate bilateral conflict, increasing the likelihood of war. They also complement the more structural evidence presented in Seitz et al. (2015) that a decrease in trade costs between two countries reduces their military spending and that reduces military spending by other countries.

Our analysis could be extended (e.g., along the lines of Eaton and Kortum, 2002) to consider multiple commodities traded among multiple trading partners. While capturing the direct and cross-price effects of security policies, this modeling choice would enable us to expand the sets of countries to explore, for example, the theoretical implications of trade among countries with no rivals for arming. As such, we could scrutinize our finding of a positive and robust correlation between trade and arming for such countries. This extension could incorporate the presence of nontraded sectors, thereby mapping more closely to reality. Equally important, by admitting the presence of multiple channels of influence between trade and arming, this richer setting would be more suitable for structural estimation. As such, it could complement nontrivially the analysis of Acemoglu and Yared (2010), who emphasize the effects of national military spending on trade, by studying the two-way causality between arming and trade. Last, but not least, this modification could allow us to explore the effects of bilateral interventions (e.g., sanctions).

The analysis could also be extended to study how security policies matter for trade agreements. In contrast to standard analyses that typically focus on tariffs set below their non-cooperative Nash levels (Bagwell and Staiger, 2001), one could consider trade negotiations in which the relevant threat-points coincide with the non-cooperative Nash equilibrium in security and trade policies. This approach could also shed light on the possible advantages of policy linkage, including international transfers and sanctions (Maggi, 2016).

Another potentially fruitful avenue for future research concerns the political economy implications of the model. In our two-country model, security policies can have different welfare implications for different factor owners when intermediate-goods production uses labor and capital in different proportions than guns production. Thus, one might consider the possibility that factor owners can influence, through lobbying or voting, the formulation of security policies. Such an extension will likely offer new insights on how differing domestic interests could affect the domestic distribution of income as well as external conflict.

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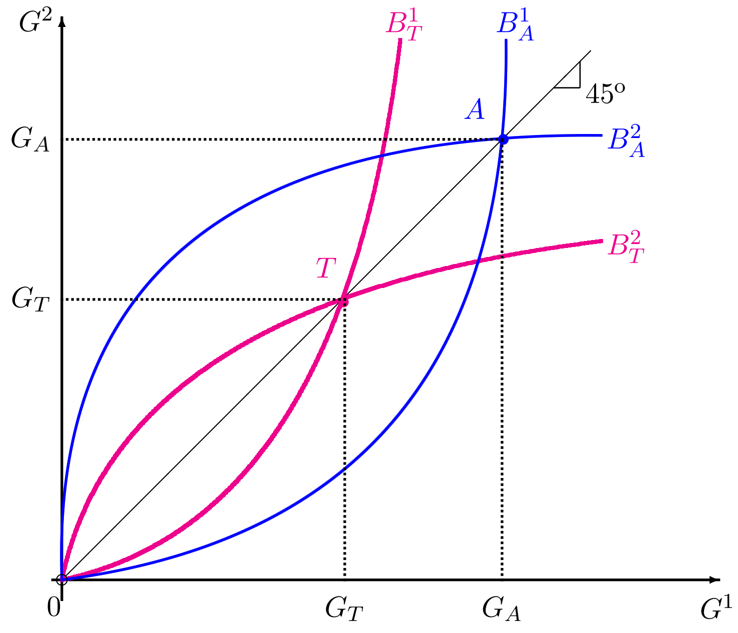
(a) Complete symmetry



(b) Extreme asymmetry in resources endowments

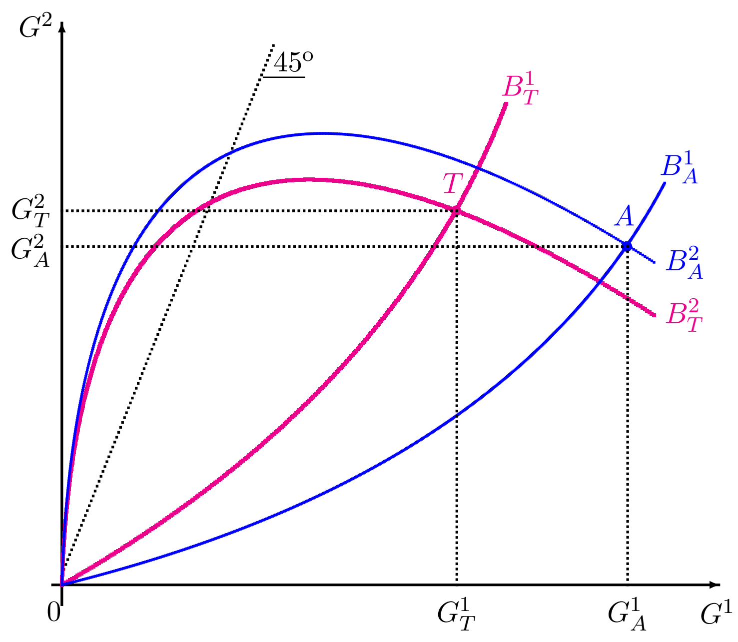


Figure 1: The Impact of Free Trade on Equilibrium Arming

Table 1: International Trade and Military Spending

	(1) CORR	(2) CNTRLS	(3) FES	(4) LAGS	(5) IVFES
TRADE_RIVALS	-0.266	-0.150	-0.122		-0.235
	$(0.031)^{**}$	$(0.032)^{**}$	$(0.036)^{**}$		$(0.076)^{**}$
TRADE_FRIENDS	1.034	0.311	0.218		0.619
	$(0.037)^{**}$	$(0.050)^{**}$	$(0.086)^{*}$		$(0.158)^{**}$
TRADE_NR	0.803	0.192	0.172		0.486
	$(0.024)^{**}$	$(0.046)^{**}$	$(0.079)^{*}$		$(0.132)^{**}$
GDP		1.322	0.091	-0.117	-0.192
		$(0.076)^{**}$	(0.187)	(0.182)	(0.255)
PPLN		-0.526	-0.044	-0.117	0.190
		$(0.056)^{**}$	(0.392)	(0.444)	(0.549)
INSTITUTIONS		-0.043	-0.035	-0.032	-0.030
		$(0.008)^{**}$	$(0.008)^{**}$	$(0.006)^{**}$	$(0.007)^{**}$
HOSTILITY		2.052	0.859	0.827	0.792
		$(0.331)^{**}$	$(0.361)^{*}$	$(0.377)^{*}$	$(0.469)^{+}$
RIVALS		0.131	0.115	0.127	-0.116
		(0.342)	(0.338)	(0.123)	(0.576)
LAG_TRADE_RIVALS				-0.067	
				$(0.039)^{+}$	
LAG_TRADE_FRIENDS				0.338	
				$(0.087)^{**}$	
LAG_TRADE_NR				0.320	
				$(0.080)^{**}$	
Year Fixed Effects	No	No	Yes	Yes	Yes
Country Fixed Effects	No	No	Yes	Yes	Yes
R^2	0.699	0.798	0.978	0.981	0.978
UnderId					30.998
χ^2 p-val					(0.000)
Weak Id					8.038
χ^2 p-val					(0.000)
Over Id					3.964
χ^2 p-val					(0.265)
N	716	716	716	654	654

Notes: This table reports results from a series of specifications, based on (18), that quantify the impact of trade on military spending. The dependent variable is always the logarithm on national military spending and all estimates are obtained with the OLS estimator. All specifications distinguish between the impact of the exports of countries that have rivals to their rivals (*TRADE_RIVALS*) or to their friends (*TRADE_FRIENDS*). The estimates in column (1) are correlations obtained without any controls and without any fixed effects. The estimates in column (2) introduce additional controls as described in the main text. Column (3) uses year and country fixed effects. Column (4) reproduces the specification from column (3) but with lagged trade variables instead. Finally, column (5) implements an instrumental variable estimator. Robust standard errors are reported in parentheses. $^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$.

A Appendix

This appendix presents proofs of the propositions and lemmas in the main text. Additional results and more technical details can be found in Supplementary Appendix B.

Lemma A.1 *For any given feasible quantities of guns and gross factor endowments, the equilibrium wage-rental ratio ω^i is independent of the prevailing trade regime (autarky or trade). Furthermore,*

(a) $\partial Z^i(\omega^i,\cdot)/\partial\omega^i = 0$ *and* $\partial^2 Z^i(\omega^i,\cdot)/\partial(\omega^i)^2 > 0$*, s.t.* $\omega^i = \arg\min_{\omega^i} Z^i(\omega^i,\cdot)$;

(b) $\partial\omega^i/\partial K_g^i > 0$, $\partial\omega^i/\partial L_g^i < 0$*, and* $\partial\omega^i/\partial G^i \gtreqless 0$ *if* $c_r^i/c_w^i \gtreqless \psi_r^i/\psi_w^i$;

(c) $\partial Z^i/\partial K_g^i > 0$ *and* $\partial^2 Z^i/(\partial K_g^i)^2 < 0$;

(d) $\partial Z^i/\partial L_g^i > 0$ *and* $\partial^2 Z^i/(\partial L_g^i)^2 < 0$;

(e) $\partial Z^i/\partial G^i < 0$ *and* $\partial^2 Z^i/(\partial G^i)^2 < 0$ *for fixed* K_g^i.

Proof: See Supplementary Appendix B.1.

Proof of Proposition 1. Let $\overline{G}^i$ be the quantity of guns country i would produce if all of L^i and K^i were employed in that sector. Assuming $f(0)$ is arbitrarily close to 0 implies that $\lim_{G^i\to 0} f'(G^i)$ and thus $\lim_{G^i\to 0}\phi_{G^i}^i$ in (12) are arbitrarily large for any $G^j \geq 0$. Therefore, $\partial V_A^i/\partial G^i > 0$ as $G^i \to 0$. Furthermore, by the definition of $\overline{G}^i$ and our assumption that both factors are essential in the production of Z^i, $V_A^i(\overline{G}^i, G^j) < V_A^i(G^i, G^j)$ for all $G^i < \overline{G}^i$ that imply $Z^i > 0$; therefore, $\partial V_A^i/\partial G^i < 0$ for a sufficiently large $G^i \in [0, \overline{G}^i]$. The continuity of V_A^i in G^i, then, implies that there exists a best-response function for each country i, $G^i = B_A^i(G^j) \in (0, \overline{G}^i)$, such that (12) holds as an equality.

Existence. As in Garfinkel et al. (2015), to establish existence it suffices to show $\partial^2 V_A^i/\left(\partial G^i\right)^2 < 0$ at $G^i = B_A^i(G^j)$. In the proof of Lemma A.1 (presented in Supplementary Appendix (B.1)), we establish the following effects of marginal changes in w^i, K_g^i and G^i on Z_w^i:

$$\begin{aligned}
Z_{ww}^i &= -(c_{ww}^i Z^i + \psi_{ww}^i G^i)/c^i > 0 \\
Z_{wK}^i &= -r^i c_w^i/(c^i)^2 < 0 \\
Z_{wG^i}^i &= -(\psi_w^i c^i - c_w^i \psi^i)/(c^i)^2.
\end{aligned}$$

Then, while keeping r^i fixed in the background (thus attributing any implied changes in ω^i to changes in w^i) and using the FOC associated with B_A^i in (12), an application of the implicit function theorem to the envelope condition $Z_\omega^i(\omega^i,\cdot) = 0$ shows:

$$w_{G^i}^i\big|_{G^i = B_A^i} = -\frac{K_0 Z_{wK}^i \phi_{G^i}^i + Z_{wG^i}^i}{Z_{ww}^i} = -\frac{\psi_w^i}{c_{ww}^i Z^i + \psi_{ww}^i G^i} > 0. \tag{A.1}$$

Since $\psi_w^i > 0$, the concavity of the unit cost functions in factor prices (i.e., $c_{ww}^i, \psi_{ww}^i < 0$) implies, in turn, that an increase in G^i in the neighborhood of B_A^i increases the country's market-clearing (relative) wage, regardless of factor intensities.

Differentiating (12) with respect to G^i, after simplifying and using (A.1), shows

$$\left.\frac{\partial^2 V_A^i}{(\partial G^i)^2}\right|_{G^i=B_A^i} = \frac{m_A^i}{c^i}\left[r^i K_0 \phi_{G^iG^i}^i + \frac{\left(\psi_w^i\right)^2}{c_{ww}^i Z^i + \psi_{ww}^i G^i}\right] < 0. \tag{A.2}$$

The negative sign of (A.2) follows from the concavity of the conflict technology in G^i (i.e., $\phi_{G^iG^i}^i < 0$) and equation (A.1). The strict quasi-concavity of V_A^i in G^i, in turn, implies the existence of an interior equilibrium.

Uniqueness. To prove uniqueness of equilibrium, it suffices to show that

$$|J| \equiv \frac{\partial^2 V_A^i}{(\partial G^i)^2}\frac{\partial^2 V_A^j}{(\partial G^j)^2} - \frac{\partial^2 V_A^i}{\partial G^i \partial G^j}\frac{\partial^2 V_A^j}{\partial G^j \partial G^i} > 0, \tag{A.3}$$

at any equilibrium point or, equivalently,

$$\frac{\partial B_A^i}{\partial G^j}\frac{\partial B_A^j}{\partial G^i} < 1, \text{ where } \quad \frac{\partial B_A^i}{\partial G^j} = -\frac{\partial^2 V_A^i}{\partial G^i \partial G^j} \Bigg/ \frac{\partial^2 V_A^i}{\left(\partial G^i\right)^2} \quad \text{for } i \neq j = 1, 2.$$

From the existence part of this proof, the sign of $\partial B_A^i/\partial G^j$ is determined by the sign of $\partial^2 V_A^i/\partial G^i \partial G^j$. To proceed, apply the implicit function theorem to $Z_\omega^i(\omega^i, \cdot) = 0$ to find

$$w_{G^j}^i\big|_{G^i=B_A^i} = -\frac{K_0 Z_{wK}^i \phi_{G^j}^i}{Z_{ww}^i} = -\frac{r^i K_0 \phi_{G^j}^i c_w^i}{c^i(c_{ww}^i Z^i + \psi_{ww}^i G^i)} < 0, \tag{A.4}$$

where the negative sign follows from the facts that $c_w^i > 0$, $c_{ww}^i, \psi_{ww}^i < 0$, and $\phi_{G^j}^i < 0$. Differentiating (12) with respect to G^j, while using (A.4), gives

$$\left.\frac{\partial^2 V_A^i}{\partial G^i \partial G^j}\right|_{G^i=B_A^i} = \frac{m_A^i}{c^i}\left[r^i K_0 \phi_{G^iG^j}^i + \frac{r^i K_0 \phi_{G^j}^i c_w^i \psi_w^i}{c^i(c_{ww}^i Z^i + \psi_{ww}^i G^i)}\right]. \tag{A.5}$$

The second term inside the square brackets is positive by (A.4). From (1), the first term is non-negative for all $B_A^i(G^j) \geq G^j$, implying that the expression above is positive, and thus G^i depends positively on G^j. However, G^i will depend negatively on G^j if $B_A^i(G^j)$ is sufficiently smaller than G^j. Thus, depending on fundamentals, two possibilities arise: (i) $\partial B_A^i/\partial G^j < 0$ and $\partial B_A^j/\partial G^i \geq 0$ for $i \neq j = 1, 2$; or (ii) $\partial B_A^i/\partial G^j > 0$ for both $i \neq j = 1, 2$. However, $|J| > 0$ in case (i), implying we need to consider only case (ii).

In the spirit of Jones (1965), let $\theta_{KZ}^i \equiv r^i c_r^i/c^i$ and $\theta_{LZ}^i \equiv w^i c_w^i/c^i$ denote respectively the cost shares of K^i and L^i in producing Z^i. Similarly let $\theta_{KG}^i \equiv r^i \psi_r^i/\psi^i$ and $\theta_{LG}^i \equiv w^i \psi_w^i/\psi^i$ denote the corresponding cost shares in the production of G^i. In addition, let $\sigma_Z^i \equiv c^i c_{wr}^i/c_w^i c_r^i$ and $\sigma_G^i \equiv \psi^i \psi_{wr}^i/\psi_w^i \psi_r^i$ denote the elasticities of substitution between factor inputs in sectors Z^i and G^i respectively. Lastly, define

$$\lambda^i \equiv \frac{\theta^i_{LG}\psi^i G^i}{\theta^i_{LG}\psi^i G^i \theta^i_{KG}\sigma^i_G + \theta^i_{LZ}c^i Z^i \theta^i_{KZ}\sigma^i_Z} > 0. \tag{A.6}$$

Applying the above definitions and country i's FOC (12), while using the linear homogeneity of c^i and ψ^i, allows us to rewrite (A.2) and (A.5) as

$$\left.\frac{\partial^2 V^i_A}{(\partial G^i)^2}\right|_{G^i=B^i_A} = -\frac{m^i_A \psi^i \phi^i_{G^i}}{c^i \phi^i}\left[-\frac{\phi^i \phi^i_{G^i G^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LG}\right] \tag{A.7a}$$

$$\left.\frac{\partial^2 V^i_A}{\partial G^i \partial G^j}\right|_{G^i=B^i_A} = -\frac{m^i_A \psi^i \phi^i_{G^j}}{c^i \phi^i}\left[-\frac{\phi^i \phi^i_{G^i G^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ}\right]. \tag{A.7b}$$

Combining these equations gives $\partial B^i_A/\partial G^j = (-\phi^i_{G^j}/\phi^i_{G^i})H^i$, where

$$H^i \equiv \frac{H^i_1}{H^i_2} \equiv \left[-\frac{\phi^i \phi^i_{G^i G^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ}\right] \bigg/ \left[-\frac{\phi^i \phi^i_{G^i G^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LG}\right]. \tag{A.8}$$

Since $(-\phi^i_{G^j}/\phi^i_{G^i})(-\phi^j_{G^i}/\phi^j_{G^j}) = 1$, we have $(\partial B^i_A/\partial G^j)(\partial B^j_A/\partial G^i) = H^i H^j$. Given our focus on the case of strategic complements (which implies $H^i > 0$ for $i = 1, 2$), proving $|J| > 0$ requires only that $H^i < 1$. In Supplementary Appendix (B.1), we show that a sufficient (but hardly necessary) condition is that σ^i_G and/or σ^i_Z are not too much smaller than one. ||

Proof of Proposition 2. Assuming zero trade costs, the world relative price of country i's imported good is identical to the domestic relative price: p^i_T. While p^i_T is endogenously determined as a function of G^i and G^j, it must satisfy $p^i_T = p^i_T \in [1/\alpha^j, \alpha^i]$. Our proof to follow abstracts from the boundary conditions on p^i_T. As such, it demonstrates the existence only of a local optimum for each country i, given G^j.[57] In Supplementary Appendix (B.1), we return to this issue, showing how weak comparative advantage (i.e., low values of α^i) matters and identifying the existence of a critical value of α^i for each country i, denoted by α^i_{0T} (> 1), such that $\alpha^i > \alpha^i_{0T}$ for both i ensures that the boundary constraints on p^i_T do not bind in the equilibrium identified here.

Let $\widetilde{V}^i_T(G^i, G^j)$ denote the unconstrained value function for country i under free trade (i.e., abstracting from the limits on p^i_T), and recall our definition of $\overline{G}^i$, as the maximum quantity of arms produced by country i using all of K^i and L^i. Assuming both labor and capital are essential in producing Z^i, $\widetilde{V}^i_T(\overline{G}^i, G^j) < \widetilde{V}^i_T(G^i, G^j)$ for all $G^i < \overline{G}^i$ that imply

[57] Our abstraction can be thought of as assuming that $\alpha^i \to \infty$ for $i = 1, 2$, which effectively reduces the model to one in which each country produces a nationally differentiated good, as is the case of the Armington (1969) model. Although convenient for ruling out possible discontinuities in the best-response functions, this simplification fails to capture the rich welfare implications we identify in our modified Ricardian model.

$Z^i > 0$. Therefore, $\partial\widetilde{V}_T^i/\partial G^i < 0$ for sufficiently large $G^i \in [0, \overline{G}^i]$. If $\sigma \leq \overline{\sigma}^i \equiv \gamma_j/\zeta^j$ evaluated at $G^i = G^j = 0$ so that the marginal benefit of arming is non-positive at $G^i = 0$, the conflict technology (1) implies $\widetilde{V}_T^i(G^i, G^j)$ reaches a peak in the domain $[0, \overline{G}^i]$ at $G^i = 0$ for $G^j \geq 0$.[58] Alternatively, if $\sigma > \overline{\sigma}^i$ so that the marginal benefit of arming is strictly positive at $G^i = 0$, then $\widetilde{V}_T^i(G^i, G^j)$ reaches a peak in the interior of the domain. In this case, equation (1) and the continuity of $\widetilde{V}_T^i(G^i, G^j)$ in G^i imply the existence of an unconstrained best-response function for country i (i.e., ignoring the constraints on p_T^i), denoted by $\widetilde{B}_T^i(G^j)$, such that the FOC (14) holds with equality.

Without loss of generality, assume that $\overline{\sigma}^1 < \overline{\sigma}^2$. Then, since the above holds true for both countries $i = 1, 2$, we have three possibilities to consider:

(i) $\sigma \leq \overline{\sigma}^1$, which implies $\partial\widetilde{V}_T^i/\partial G^i|_{G^i=0} < 0$ for $i = 1, 2$;
(ii) $\sigma \in (\overline{\sigma}^1, \overline{\sigma}^2]$ which implies $\partial\widetilde{V}_T^1/\partial G^1|_{G^1=0} < 0$, while $\partial\widetilde{V}_T^2/\partial G^2|_{G^2=0} > 0$;
(iii) $\sigma > \overline{\sigma}^2$, which implies $\partial\widetilde{V}_T^i/\partial G^i|_{G^i=0} > 0$ for $i = 1, 2$.

In case (i), there exists an equilibrium where each country i chooses $G_T^i = 0$. Similarly, $G_T^i = 0$ in case (ii) for country i that has $\partial\widetilde{V}_T^i/\partial G^i < 0$ at $G^i = 0$ for $G^j = 0$. By contrast, in case (ii) for the other country $j \neq i$ and in case (iii) for both countries i, the marginal net benefit from producing arms, when evaluated at zero arming, is strictly positive. Hence, to establish the existence of a pure-strategy equilibrium in security policies under trade $(G_T^1, G_T^2) \neq (G_A^1, G_A^2)$, assuming that α^i is arbitrarily large for both i, we need to prove only that $\widetilde{V}_T^i(G^i, G^j)$ is strictly quasi-concave in G^i for country i when $\partial\widetilde{V}_T^i/\partial G^i = 0$ at $G^i = \widetilde{B}_T^i(G^j) > 0$ given $G^j \geq 0$, in cases (ii) and (iii).[59] We present this part of the proof in Supplementary Appendix (B.1). ||

Proof of Lemma 2. To confirm that a shift from autarky to free trade induces a positive welfare effect that exceeds the traditions gains from trade (i.e., given guns) provided trade induces lower arming, we decompose the changes in payoffs into three parts. First, starting from the equilibrium under autarky, allow country j to reduce its guns from G_A^j to $G_T^j \geq 0$. For such moves along $B_A^i(G^j)$, only the strategic effect on country i's payoff matters:

$$\frac{1}{V_A^i}\frac{\partial V_A^i}{\partial(-G^j)}\bigg|_{G^i=B_A^i(G^j)} = -\frac{dZ^i/dG^j}{Z^i} > 0,$$

which implies $V_A^i(B_A^i(G_T^j), G_T^j) > V_A^i(G_A^i, G_A^j)$. Second, consider the welfare effect of a shift to trade, given $G^i = B_A^i(G_T^j)$ and $G^j = G_T^j$. From Lemma 1, such a shift implies

[58] As suggested in the main text, we cannot rule out the possibility that there exists another local maximum for $G^i \in (0, \overline{G})$.

[59] Note that, by the continuity of our specification of $\phi^i = \phi(G^i, G^j)$ in (1) at $G^1 = G^2 = 0$, with an appropriate parameterization of this conflict technology (e.g., in terms of the parameters δ and b for the example shown in footnote 16), we can ensure that $\sigma - \gamma_2/\zeta^2$ remains negative when evaluated at country 2's best response to $G^1 = 0$ for case (ii).

a non-negative welfare effect: $V_T^i(B_A^i(G_T^j), G_T^j) \geq V_A^i(B_A^i(G_T^j), G_T^j)$. Third, country i's shift to trade induces it to adjust its arming, from $B_A^i(G^j)$ to $B_T^i(G^j)$ given $G^j = G_T^j \geq 0$. This adjustment also produces a non-negative welfare effect, since $V_T^i(B_T^i(G_T^j), G_T^j) \geq V_T^i(B_A^i(G_T^j), G_T^j)$. Bringing these results together implies $V_T^i(G_T^i, G_T^j) > V_A^i(G_A^i, G_A^j)$ for $i \neq j = 1, 2$. Since the first part of our decomposition is strictly positive (under the assumption that arming by the opponent falls under trade relative to autarky) and the third part is non-negative, each country's gain will exceed those predicted by models that abstract from conflict as captured by the second part.

Finally, observe that the proof of Proposition 2 indicates country i's payoff under free trade is comparable to its payoff under autarky for any given $G^j \geq 0$, provided that α^i is sufficiently large. Thus, we need to consider only circumstances where the lower price constraint $p_T^i \geq 1/\alpha^j$ binds for country i. Fix G^j at some level, and suppose that $p_T^i = 1/\alpha^j$ for some value of G^i—call it G_c^i. Because the unconstrained optimal value of G^i level is less than G_c^i, we know that $\widetilde{V}_T^i$ and thus V_T^i rise as G^i approaches G_c^i from above. But, V_T^i also rises as G^i approaches G_c^i from below, since the optimal value of G^i at constant prices equals $B_A^i(G^j)$ which exceeds G_c^i, given G^j. Thus, V_T^i reaches a (kinked) peak at G_c^i. ||

Proof of Lemma 3. Using (15) with $\widehat{\tau}^i = \widehat{\tau}^j = \widehat{t}^i = 0$, one can easily verify that the following shows country i's incentive to arm under trade given his rival's arming G^j:

$$\frac{\partial V_T^i/\partial G^i}{V_T^i} = \left(1 - \rho^i\right) \frac{dZ^i/dG^i}{Z^i} + \rho^i \frac{dZ^j/dG^i}{Z^j},$$

where as previously defined, $\rho^i \equiv (\gamma_j^i/t^i\Delta)[\left(t^i - 1\right)\varepsilon^i + 1]$. Maintaining focus on the case where $\sigma > 1$, we have $\rho^i \in (0, 1)$. Next evaluate the above at the solution for arming by country i under autarky, as implicitly defined by the FOC implied by (12) or equivalently where $dZ^i/dG^i = 0$. Since $\rho^i > 0$ and from (11b) we have $dZ^j/dG^i < 0$, the above expression evaluated at G_A^i given any feasible G^j is necessarily negative. (Equations (B.17) and (B.18) presented in Supplementary Appendix B.2 show how the parameter ρ^i simplifies respectively in the cases of (i) geographic trade costs only and (ii) tariffs chosen non-cooperative and simultaneously with security policies.) ||

Proof of Proposition 3.

Part a. That at least one country and possibly both arm less under trade than under autarky follows from the conflict technology (1) and Lemma 3. In Supplementary Appendix B.2, we provide numerical examples confirming the possibility that one country arms by more under trade, when the international distribution of secure resources is extremely uneven.

Part b. As discussed in the proof of Proposition 1, $\partial B_A^i/\partial G^j > 0$ for all $G^j \leq G_A^j$; but, one country i's best-response function could be negatively related to its rival's guns

G^j in the neighborhood of the autarkic equilibrium. Given that and drawing from the analysis of Section 5, we distinguish between two cases under autarky. (i) $\partial B_A^i/\partial G^j > 0$ for $i \neq j = 1, 2$, which arises when the mix of secure resource endowments of labor and capital are sufficiently even across countries; and, (ii) $\partial B_A^i/\partial G^j > 0$ for all $G^j \leq G_A^j$ while $\partial B_A^j/\partial G^i < 0$ for G^i close to G_A^i, which arises when one country i has a sufficiently greater amount of secure labor to land relative to its rival j, such that $G_A^i > G_A^j$.

The proof of this part of the proposition also builds on Lemma 3 that shows $B_T^i(G^j) < B_A^i(G^j)$ for $i \neq j = 1, 2$. In case (i) where neither country's arming is a strategic substitute for its rival's arming in the neighborhood of the autarkic equilibrium, the just-described shifts in each country's best-response functions in a move to trade necessarily result in lower equilibrium spending by both countries. Since $G_T^i < G_A^i$ for both countries i, aggregate equilibrium arming must fall.

To show that the proposition holds in case (ii) as well, we first prove that $\partial B_A^2/\partial G^1 + 1 > 0$ for all (G^1, G^2) that satisfy $\partial B_A^2/\partial G^1 < 0$ and $\partial B_A^1/\partial G^2 > 0$. To this end, first recall from the proof of Proposition 1 that $\partial B_A^i/\partial G^j = (-\phi^i_{G^j}/\phi^i_{G^i})H^i$, where H^i is given by (A.8). Economizing on notation, let $\Phi \equiv \frac{\phi^2\lambda}{\phi^2_{G^2}G^2} > 0$, $x \equiv \theta^2_{LZ} > 0$ and $y \equiv \theta^2_{LG} > 0$. Then,

$$\partial B_A^2/\partial G^1 = \left(-\frac{\phi^2_{G^1}}{\phi^2_{G^2}}\right)\left(\frac{-\frac{\phi^2\phi^2_{G^2G^1}}{\phi^2_{G^1}\phi^2_{G^2}} + Ax}{-\frac{\phi^2\phi^2_{G^2G^2}}{\phi^2_{G^2}\phi^2_{G^2}} + Ay}\right).$$

With the specification of ϕ^i in (1), we can rewrite the above equation as

$$\partial B_A^2/\partial G^1 = \left[\frac{f(G^2)f'(G^1)}{f(G^1)f'(G^2)}\right]\left(\frac{\frac{f(G^2)}{f(G^1)} + Ax - 1}{2\frac{f(G^2)}{f(G^1)} - \frac{f(G^2)f''(G^2)}{(f'(G^2))^2\phi^1} + Ay}\right),$$

where $f(\cdot) > 0$, $f'(\cdot)$ and $f''(\cdot) \leq 0$. Since the denominator of the second (multiplicative) term is positive, the sign of $\partial B_A^2/\partial G^1 + 1$ coincides with the sign of the following

$$\begin{aligned} C &= \left[\frac{f(G^2)\, f'(G^1)}{f(G^1)\, f'(G^2)}\right]\left[\frac{f(G^2)}{f(G^1)} + Ax - 1\right] + 2\frac{f(G^2)}{f(G^1)} - \frac{f(G^2)\, f''(G^2)}{(f'(G^2))^2\,\phi^1} + Ay \\ &= \frac{f(G^2)}{f(G^1)}\left[2 - \frac{f'(G^1)}{f'(G^2)}\right] + \left[\frac{f(G^2)\, f'(G^1)}{f(G^1)\, f'(G^2)}\right]\left[\frac{f(G^2)}{f(G^1)} + Ax\right] + Ay - \frac{f(G^2)\, f''(G^2)}{(f'(G^2))^2\,\phi^1}. \end{aligned}$$

The last two terms of the second line are clearly non-negative. Furthermore, the concavity of $f(\cdot)$ and the assumption that $G^1 > G^2$ together imply $f'(G^1)/f'(G^2) \leq 1$. Thus, $C > 0$, and $\partial B_A^2/\partial G^1 + 1 > 0$.

Starting from any value of G_0^1 on the downward sloping segment of country 2's best-response function, fix the associated sum $G_0^1 + B_A^2(G_0^1) = G^1 + G^2$ and consider a marginal decrease in G^1. The result that $\partial B_A^2/\partial G^1 + 1 > 0$ at $G^1 = G_0^1$ implies the decrease in G^1 below G_0^1 necessarily raises B_A^2 by less than the increase in G^2 needed to keep $G^1 + G^2$ constant at $G_0^1 + B_A^2(G_0^1)$—i.e., $B_A^2(G^1) < G^2$ for $G^1 = G_0^1 - \epsilon$, where $\epsilon > 0$. It then follows that $B_A^2(G^1) < G_A^2$ for any $G^1 < G_A^1$ and G^2 such that $G^1 + G^2 = G_A^1 + G_A^2$, which implies in turn that $G^1 + B_A^2(G^1) < G^1 + G^2 = G_A^1 + G_A^2$. But, since $B_T^2(G^1) < B_A^2(G^1)$, we must have $G^1 + B_T^2(G^1) < G_A^1 + G_A^2$. Finally, let $G^1 = G_T^1$, which implies $G^1 + B_T^2(G^1) = G_T^1 + G_T^2 < G_A^1 + G_A^2$, thereby completing the proof. $||$

Proof of Proposition 4.

Part a. Assume a unique, interior equilibrium in security policies exists under free trade. Given our assumption that the two contending countries are identical in all respects means that, when $G^i = G$, $Z^1 = Z^2$, so that $\nu^i = \frac{1}{2}$, $w^i = w$, $r^i = r$, and $c^i = c$ for $i = 1, 2$. Provided the two consumption goods are sufficiently substitutable to ensure positive marginal costs of arming ($\sigma - \frac{1}{2}\gamma_2 > 0$), the FOCs under trade (17) can be written as

$$\text{sign}\left\{ \frac{\partial V_T^i}{\partial G^i}\bigg|_{G^i=G} \right\} = \text{sign}\left\{ ArK_0 \phi_{G^i}^i|_{G^i=G} - \psi \right\},$$

for $i = 1, 2$, where $A = 1 + \frac{\gamma_2/2\sigma}{1-\gamma_2/2\sigma} > 1$, for $\sigma < \infty$. Recall that the FOC for country i's arming choice under autarky (12) requires $dZ^i/dG^i = 0$ or equivalently $rK_0\phi_{G^i}^i|_{G^i=G_A^*} - \psi = 0$. Since $A > 1$, the sign of the FOC under trade when evaluated at $G^i = G_A$ for $i = 1, 2$ is strictly positive. The result then follows from the implicit function theorem.

Part b: To prove this part, we focus on cases where the strength of comparative advantage (i.e., $\alpha^i = a_2 > 1$ for $i = 1, 2$) is sufficiently small to imply that world market-clearing price is very close to the autarky price, such that the gains from trade are very small. Since the increased security costs implied by part (a) are independent of α^i, they can easily dwarf the relatively small gains from trade. See Supplementary Appendix B.1 for technical details. $||$

B Supplementary Appendix (For Online Publication Only)

This supplementary appendix has two main sections. The first section (B.1) provides a proof of Lemma A.1 and presents the more technical details for the proofs presented in Appendix A. It also contains an additional proposition regarding the effect of the elasticity of substitution on equilibrium arming in the case of complete symmetry and its proof. The second section (B.2) extends the analysis of two countries where trade costs are present, in support of the analysis in Section 5.3, and presents some additional results based on numerical methods.

B.1 Additional Technical Details

Proof of Lemma A.1. That the equilibrium wage-rental ratio ω^i is independent of the trade regime in place, for given gross endowments and guns, follows directly from the equilibrium condition in (4).

To establish parts (a)–(e) of the lemma, let us temporarily omit country superscripts. Recall that

$$Z(\omega, K_g, L_g, G) = \frac{wL_g + rK_g - \psi G}{c(w,r)} = \frac{\omega L_g + K_g - \psi(\omega,1)G}{c(\omega,1)}. \tag{B.1}$$

Now differentiate Z with respect to w, K_g, L_g, and G to obtain the following (after some algebra):[1]

$$\frac{\partial Z}{\partial w} \equiv Z_w = \left[\frac{rc_w}{c^2}\right](L_g - \psi_w G)\left[\frac{c_r}{c_w} - \frac{K_g - \psi_r G}{L_g - \psi_w G}\right] \tag{B.2a}$$

$$\frac{\partial Z}{\partial K_g} \equiv Z_K = r/c \tag{B.2b}$$

$$\frac{\partial Z}{\partial L_g} \equiv Z_L = w/c \tag{B.2c}$$

$$\frac{\partial Z}{\partial G} \equiv Z_G = -\psi/c. \tag{B.2d}$$

Taking derivatives of the above expressions with respect to w shows

$$Z_{ww} = -(c_{ww}Z + \psi_{ww}G)/c > 0 \tag{B.3a}$$

$$Z_{Kw} = -rc_w/c^2 < 0 \tag{B.3b}$$

$$Z_{Lw} = rc_r/c^2 > 0 \tag{B.3c}$$

$$Z_{Gw} = -\frac{\psi_w c - c_w \psi}{c^2} = \frac{r(\psi_r c_w - \psi_w c_r)}{c^2} = \frac{r\psi_w c_w}{c^2}\left(\frac{\psi_r}{\psi_w} - \frac{c_r}{c_w}\right). \tag{B.3d}$$

Part (a). As can be seen from (B.1), to trace the role of ω it is sufficient to consider the

[1] We abstract here from the dependence of K_g on guns by treating gross factor endowments as exogenous; however, we explicitly consider this dependence in our analysis of endogenous security policies.

impact of variations in w keeping r fixed in the background. The expression inside the last set of square brackets in (B.2a) coincides with the domestic market-clearing condition (4) that produces the solution for ω; therefore, $Z_w = 0$ evaluated at the value of ω (ω^e) or, equivalently, $Z_\omega(\omega^e, \cdot) = 0$. Thus, the first component of this part of the lemma follows from (B.2a). The second component follows from the sign of the expression for Z_{ww} in (B.3a), as implied by (5), (4), the linear homogeneity of the unit costs functions ψ and c, as well as their concavity in factor prices. That Z^e is strictly quasi-convex in ω and minimized at the equilibrium value implies that Z^e is the envelope function of $Z(\omega, \cdot)$, which is useful for establishing the remaining parts of the lemma and for our subsequent characterization of equilibria in security policies.

Part (b). This part follows from the implicit function theorem applied to $Z^e_w = 0$, while using (B.3). The first two components conform to intuition that an increase in the relative supply of one factor (capital or labor) decreases its relative price in factor markets. The last component reveals that, if the technology for guns is labor intensive (i.e., $c_r/c_w > \psi_r/\psi_w$), then, at constant factor prices, production of an additional gun will increase the demand for labor relative to capital, thus forcing the wage-rental ratio to rise.

Parts (c)–(e). The proof of the first component of parts (c), (d) and (e) follows readily from (B.2b)–(B.2d) with $Z^e_\omega = 0$ and the fact that $dZ^e = Z^e_\omega d\omega^e + Z^e_K dK_g + Z^e_L dL_g + Z^e_G dG$. The strict concavity of Z^e in, say, K_g can be proven as follows. First, observe that $dZ^e/dK_g = Z^e_\omega(d\omega^e/dK_g) + Z^e_K$. Now differentiate this expression with respect to K_g to find

$$\frac{d^2 Z^e}{dK_g^2} = Z^e_{\omega\omega}\left(\frac{d\omega^e}{dK_g}\right)^2 + 2Z^e_{\omega K}\frac{d\omega^e}{dK_g} + Z^e_\omega \frac{d^2\omega^e}{dK_g^2} + Z^e_{KK}.$$

Since $Z^e_\omega = 0$, $Z^e_{KK} = 0$, and $d\omega^e/dK_g = -Z^e_{\omega K}/Z^e_{\omega\omega}$, the expression above simplifies as

$$\frac{d^2 Z^e}{dK_g^2} = -\frac{(Z^e_{\omega K})^2}{Z^e_{\omega\omega}} < 0,$$

giving us the desired result. The strict concavity of Z^e in L_g and in G in parts (d) and (e) respectively can be shown along the same lines. Parts (c) and (d) state that Z^e is increasing and strictly concave in the country's gross factor endowments. Part (e) establishes that, given K_g, Z^e is decreasing and strictly concave in G. ||

Proof of Proposition 1 continued. To derive the sufficient conditions for uniqueness of the equilibrium in security policies under autarky as stated in the proposition, we demonstrate that the expression for $H^i \equiv H^i_1/H^i_2$ shown in equation (A.8) of Appendix A is less than one. Because the numerator (H^i_1) and the denominator (H^i_2) are both positive, we can subtract the former from the latter to obtain

$$C^i = H_2^i - H_1^i = -\frac{\phi^i \phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LG} - \left(-\frac{\phi^i\phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ}\right).$$

If $C^i > 0$, then $H^i < 1$ holds. The properties of $\phi(G^i, G^j)$ imply

$$\begin{aligned}
-\frac{\phi^i\phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} &= \frac{1}{\phi^j}\left[2\phi^i - \frac{f(G^i)f''(G^i)}{f'(G^i)f'(G^i)}\right] > 0 \quad (\text{since } f'' \leq 0)\\
-\frac{\phi^i\phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} &= \frac{\phi^i}{\phi^j} - 1.
\end{aligned}$$

Using the above observations in C^i after rearranging terms gives

$$C^i = \frac{1}{\phi^j}\left[\phi^i - \frac{f(G^i)f''(G^i)}{f'(G^i)f'(G^i)}\right] + \left[1 - \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ}\right] + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LG}.$$

A sufficient (but hardly necessary) condition for $C^i > 0$ is that the expression inside the second set of square brackets is non-negative. The definition of λ^i in (A.6) and the FOC under autarky (12) together imply after tedious algebra

$$\begin{aligned}
\frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ} &= \frac{r^i\phi^iK_0\lambda^i}{\psi^iG^i}\theta^i_{LZ} = \frac{\theta^i_{LG}\theta^i_{LZ}\left(r^i\phi^iK_0\right)}{\sigma^i_G\theta^i_{LG}\theta^i_{KG}\psi^iG^i + \sigma^i_Z\theta^i_{LZ}\theta^i_{KZ}c^iZ^i}\\
&= \frac{\theta^i_{LG}\theta^i_{LZ}\left(r^i\phi^iK_0\right)}{M^i + \theta^i_{LG}\theta^i_{KG}\psi^iG^i + \theta^i_{LZ}\theta^i_{KZ}c^iZ^i}. \qquad \text{(B.4)}
\end{aligned}$$

where $M^i \equiv (\sigma^i_G - 1)\theta^i_{LG}\theta^i_{KG}\psi^iG^i + (\sigma^i_Z - 1)\theta^i_{LZ}\theta^i_{KZ}c^iZ^i$. Solving for G^i and Z^i from (3a) and (3b) and using these expressions in gives:

$$\begin{aligned}
\psi^iG^i &= \frac{\theta^i_{KZ}w^iL^i - \theta^i_{LZ}r^i\left(K^i + \phi^iK_0\right)}{\theta^i_{KZ} - \theta^i_{KG}}\\
c^iZ^i &= \frac{-\theta^i_{KG}w^iL^i + \theta^i_{LG}r^i\left(K^i + \phi^iK_0\right)}{\theta^i_{KZ} - \theta^i_{KG}},
\end{aligned}$$

which together imply

$$\theta^i_{LG}\theta^i_{KG}\psi^iG^i + \theta^i_{LZ}\theta^i_{KZ}c^iZ^i = \theta^i_{KG}\theta^i_{KZ}w^iL^i + \theta^i_{LG}\theta^i_{LZ}r^i\left(K^i + \phi^iK_0\right).$$

Substitution of the above in (B.4) shows

$$\frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ} = \left[\frac{\theta^i_{LG}\theta^i_{LZ}r^i\left(K^i + \phi^iK_0\right)}{M^i + \theta^i_{KG}\theta^i_{KZ}w^iL^i + \theta^i_{LG}\theta^i_{LZ}r^i\left(K^i + \phi^iK_0\right)}\right]\left(\frac{\phi^iK_0}{K^i + \phi^iK_0}\right),$$

which is less than one, implying $C^i > 0$, $H^i < 1$ and thus uniqueness of equilibrium under autarky, if M^i is not very negative. A sufficient condition is that σ_G^i and/or σ_G^i are not too much smaller than one. ||

Proof of Proposition 2 continued.

Strict quasi-concavity of $\widetilde{V}_T^i$ in G^i. To present this proof, we introduce more compact notation. In particular, define $F^i \equiv Z^i(K_Z^i, L_Z^i)$ and let $F_{G^i}^i \equiv dZ^i/dG^i$ shown in (11a) and $F_{G^i}^j \equiv dZ^j/dG^i$ shown in (11b), for $j \neq i = 1, 2$, indicate the effects of a change in G^i on those optimized values. Then, with equation (13), we can rewrite country i's FOC under trade, focusing on interior solutions, as follows:

$$\frac{\partial \widetilde{V}_T^i}{\partial G^i} = \widetilde{V}_T^i \left[\frac{F_{G^i}^i}{F^i} - \frac{\gamma_j}{\sigma} \left(\frac{F_{G^i}^i}{F^i} - \frac{F_{G^i}^j}{F^j} \right) \right] = 0, \tag{B.6}$$

where $F_{G^i}^i/F^i - F_{G^i}^j/F^j > 0$ which implies $F_{G^i}^i/F^i > 0$. Note from (9) that $\partial p_T^i/\partial G^i = (p_T^i/\sigma)(F_{G^i}^i/F^i - F_{G^i}^j/F^j)$ and recall $\partial \gamma_j^i/\partial p_j^i = -(1-\gamma_j^i)(\sigma-1)(\gamma_j^i/p_j^i)$. Differentiation of the expression above evaluated at an interior solution yields:

$$\begin{aligned} \left.\frac{\partial^2 \widetilde{V}_T^i}{(\partial G^i)^2}\right|_{G^i = \widetilde{B}_T^i} &= \widetilde{V}_T^i \left\{ \frac{(\sigma-1)(1-\gamma_j)}{\gamma_j} \left(F_{G^i}^i/F^i\right)^2 \right. \\ &\quad + \left(1 - \frac{\gamma_j}{\sigma}\right) \left[\left(F_{G^iG^i}^i/F^i\right) - \left(F_{G^i}^i/F^i\right)^2 \right] \\ &\quad \left. + \left(\frac{\gamma_j}{\sigma}\right) \left[\left(F_{G^iG^i}^j/F^j\right) - \left(F_{G^i}^j/F^j\right)^2 \right] \right\}. \end{aligned} \tag{B.7}$$

To evaluate the sign of this expression, we apply the implicit function theorem to $Z_\omega^i(\omega^i, \cdot) = 0$, using equations (B.3) and (11) with $F^i \equiv Z^i$ and (as before) attributing any implied changes in ω^i to changes in w^i alone:

$$\begin{aligned} \left. w_{G^i}^i \right|_{G^i = \widetilde{B}_T^i} &= -\frac{K_0 Z_{wK}^i \phi_{G^i}^i + Z_{wG^i}^i}{Z_{ww}^i} = -\frac{c_w^i F_{G^i}^i + \psi_w^i}{c_{ww}^i F^i + \psi_{ww}^i G^i} > 0 \\ \left. w_{G^i}^j \right|_{G^i = \widetilde{B}_T^i} &= -\frac{K_0 Z_{wK}^j \phi_{G^i}^i}{Z_{ww}^j} = -\frac{c_w^j F_{G^i}^j}{c_{ww}^j F^j + \psi_{ww}^j G^j} < 0. \end{aligned}$$

With these expressions, one can verify the following:

$$\begin{aligned} \left. F_{G^iG^i}^i \right|_{G^i = \widetilde{B}_T^i} &= \frac{1}{c^i} \left[r^i K_0 \phi_{G^iG^i}^i + \frac{\left(c_w^i F_{G^i}^i + \psi_w^i\right)^2}{c_{ww}^i F^i + \psi_{ww}^i G^i} \right] < 0 \\ \left. F_{G^iG^i}^j \right|_{G^i = \widetilde{B}_T^i} &= \frac{1}{c^j} \left[r^j K_0 \phi_{G^iG^i}^j + \frac{\left(c_w^j F_{G^i}^j\right)^2}{c_{ww}^j F^j + \psi_{ww}^j G^j} \right]. \end{aligned}$$

Then, substitute the above into (B.7) and invoke the FOC under free trade (B.6), using the fact that $\phi^i_{G^i} = -\phi^j_{G^i}$. After rearranging, we have

$$\left.\frac{\partial^2 \widetilde{V}^i_T}{(\partial G^i)^2}\right|_{G^i=\widetilde{B}^i_T} = \widetilde{V}^i_T \left\{ -\left[\frac{1-\gamma_j+(\sigma-1)\gamma_j}{\gamma_j}\right](F^i_{G^i}/F^i)^2 + \left(\frac{1-\gamma_j/\sigma}{c^iF^i}\right)\frac{\phi^i_{G^iG^i}\psi^i}{\phi^i_{G^i}} + \left(\frac{1-\gamma_j/\sigma}{c^iF^i}\right)\frac{(c^i_wF^i_{G^i}+\psi^i_w)^2}{c^i_{ww}F^i+\psi^i_{ww}G^i} + \left(\frac{\gamma_j/\sigma}{c^jF^j}\right)\frac{(c^j_wF^j_{G^i})^2}{c^j_{ww}F^j+\psi^j_{ww}G^j}\right\} < 0. \tag{B.8}$$

The negative sign follows from our assumptions that $\sigma > \gamma_j$ (to ensure that the marginal cost of arming is strictly positive) and $\phi^i_{G^iG^i}, c^i_{ww}, \psi^i_{ww} < 0$.

Sufficiently strong comparative advantage. We now turn to the condition in the proposition that α^i is sufficiently large. As noted earlier, for any given G^j, country i's choice of guns (G^i) influences its terms of trade and thus the domestic price of country i's imported good, p^i_T, which equals p^i_T in the absence of trade costs. However, this price cannot rise above the analogous price in country i under autarky, $p^i_A = \alpha^i$; nor can it fall below the relative domestic price of that good that prevails in country j, $1/p^j_A = 1/\alpha^j$. Thus, we have $p^i_T \in \left[1/\alpha^j, \alpha^i\right]$. In the main text we refer to α^i as the strength of country i's comparative advantage in producing good i. The smaller are α^i and α^j, the smaller is the range of prices within which country i's domestic price for its imported good can lie. This limited range, in turn, can lead to the emergence of multiple peaks in each country's payoff function given the opponent's guns choice that might generate discontinuities in best-response functions and, thus, can imply the non-existence of a pure-strategy equilibrium.

Above, we analyzed the outcome in security policies (G^1_T, G^2_T) based on the countries' unconstrained best-response functions, $\widetilde{B}^i_T(G^j)$ $i = 1, 2$ that ignore these boundary conditions. The requirement for (G^1_T, G^2_T) to be an equilibrium point is that it belongs to both countries' constrained best-response functions, $B^i_T(G^j)$ for $i = 1, 2$ that take into account the boundaries on the feasible range for p^i_T. The potential problem is that depending on the value of α^i, $B^i_T(G^j)$ could consist of two segments in the neighborhood of G^j_T, one that lies on $B^i_A(G^j)$ and another that lies on $\widetilde{B}^i_T(G^j)$, as illustrated in Fig. B.1. Thus, whether (G^1_T, G^2_T) qualifies as a pure-strategy equilibrium or not hinges on the location of the discontinuity relative to G^j_T. We now show that location depends on the strength of the two countries' comparative advantage, α^i for $i = 1, 2$.

To proceed, observe that, from (7) and the fact that $p^i_A = \alpha^i$, we can write $V^i_A(G^i, G^j) = V^i_A(G^i, G^j; \alpha^i)$, where $\partial V^i_A/\partial\alpha^i < 0$; and, similarly from (10), we can write $\widetilde{V}^i_T\left(G^i, G^j\right) = V^i_T\left(p^i_T\left(G^i, G^j\right), G^i, G^j\right)$, which gives us country i's payoff under trade when it accounts for the terms-of-trade effect, without imposing the constraint that $p^i_T \leq \alpha^i$. Note especially that the strength of comparative advantage α^i has no direct effect on unconstrained payoffs under trade, $\partial \widetilde{V}_T^i/\partial \alpha^i = 0$.

Now consider a feasible value of G^j, and the following two values of α^i for country i associated with that level of arming by the rival:

(i) $\underline{\alpha}^i \equiv p_T^i(\widetilde{B}_T^i\left(G^j\right), G^j)$ denotes the value of α^i for which, given G^j, the terms-of-trade effect of country i's arming just vanishes, such that

$$V_A^i(\widetilde{B}_T^i(G^j), G^j; \underline{\alpha}^i) = \widetilde{V}_T^i(\widetilde{B}_T^i\left(G^j\right), G^j\ );$$

(ii) $\overline{\alpha}^i \equiv p_T^i(B_A^i(G^j), G^j)$ denotes the value of α^i that, given G^j, makes country i's payoff under autarky just equal to its payoff under trade if it were to operate along B_A^i, ignoring the terms-of-trade effect of its arming decision:

$$\widetilde{V}_T^i\left(B_A^i\left(G^j\right), G^j\right) = V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right).$$

Observe that $\partial p_T^i/\partial G^i > 0$ for any $G^i < B_A^i(G^j)$. In addition, comparing the FOC under autarky (12) with the FOC under trade (14) shows that $\widetilde{B}_T^i\left(G^j\right) < B_A^i(G^j)$ for any feasible G^j. Thus, we have $\underline{\alpha}^i < \overline{\alpha}^i$. Furthermore, since $\partial V_A^i/\partial \alpha^i < 0$, $V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right)$ holds. Moreover, we have

$$V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right) > V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right).$$

The definition of $\underline{\alpha}^i$ and the fact that $V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > V_A^i(\widetilde{B}_T^i(G^j), G^j; \underline{\alpha}^i)$ gives the first inequality. Intuitively, when $\alpha = \underline{\alpha}^i \equiv p_T^i(\widetilde{B}_T^i\left(G^j\right), G^j))$, there are no gains from trade and prices are fixed, so that country i improves its payoffs by operating on its best-response function under autarky $B_A^i(G^j)$ and thus increasing G^i all the way to $B_A^i(G^j)$. The validity of the second inequality follows from the definition of $\overline{\alpha}^i$ and the fact that $\widetilde{V}_T^i(\widetilde{B}_T^i\left(G^j), G^j\right) > \widetilde{V}_T^i\left(B_A^i\left(G^j\right), G^j\right)$. When $\alpha^i = \overline{\alpha}^i \equiv p_T^i(B_A^i(G^j), G^j)$, country i can improve its payoff by operating on its best-response function under trade, $\widetilde{B}_T^i\left(G^j\right)$, which takes into account the terms-of-trade effect of arming. Since V_A^i is continuously decreasing in α^i, there exists a value of α^i, $\alpha_0^i \in (\underline{\alpha}^i, \overline{\alpha}^i)$, that solves

$$V_A^i\left(B_A^i\left(G^j\right), G^j; \alpha_0^i\right) = \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right).$$

Note that this value of α_0^i generally depends on G^j. Furthermore, there exist combinations of G^i and G^j such that $p_T^i(G^i, G^j) = \alpha_0^i$.

Let $\Lambda^i\left(G^j, \alpha\right)$ denote the difference in payoffs when country i operates on $\widetilde{B}_T^i(G^j)$ and when it operates on $B_A^i(G^j)$ given α^i and G^j, for $i \neq j = 1, 2$:

$$\Lambda^i\left(G^j, \alpha^i\right) \equiv \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right) - V_A^i\left(B_A^i\left(G^j\right), G^j; \alpha^i\right).$$

This function helps pin down possible discontinuities in player i's constrained best-response function under free trade, $B^i_T\left(G^j;\alpha^i\right)$, and clarifies their dependence on α^i. For any given G^j and α^i, we have

$$B^i_T(G^j;\alpha^i) = \begin{cases} \widetilde{B}^i_T(G^j) & \text{if} \quad \Lambda^i\left(G^j,\alpha^i\right) \geq 0 \\ B^i_A(G^j) & \text{if} \quad \Lambda^i\left(G^j,\alpha^i\right) \leq 0. \end{cases}$$

Since $\partial V^i_A/\partial\alpha^i < 0$ and $\widetilde{V}^i_T$ is independent of α^i, $\partial\Lambda^i/\partial\alpha^i \equiv \Lambda^i_\alpha > 0$. Of course, the sign of $\Lambda^i\left(G^j,\alpha\right)$ also depends on G^j. With the envelope theorem, the negative externality that the rival's arming confers on country i implies $\partial\widetilde{V}^i_T/\partial G^j < 0$ and $\partial V^i_A/\partial G^j < 0$ for feasible G^j. Thus, unless we impose additional structure on the model, we cannot sign $\partial\Lambda^i/\partial G^j \equiv \Lambda^i_{G^j}$. $\Lambda^i\left(G^j,\alpha\right)$ could change signs (positive to negative or vice versa) or not change at all as G^j changes. This last possibility means that the equality $\Lambda^i\left(G^j,\alpha^i_0\right) = 0$ need not imply $B^i_T(\cdot)$ is discontinuous at G^j given $\alpha^i = \alpha^i_0$.[2]

Nevertheless, assuming $\Lambda^i_{G^j} \neq 0$ so that $\Lambda^i\left(G^j,\alpha^i_0\right) = 0$ does imply a discontinuity in $B^i_T(G^j)$, we can be more precise about how values of α^i for $i = 1,2$ matter for the existence of a pure-strategy equilibrium under trade. Suppose $G^j = G^j_T$, and find the value of α^i (denoted by α^i_{0T}) that implies $\Lambda^i(G^j_T,\alpha^i_{0T}) = 0$. Since $\Lambda^i_\alpha > 0$, we know that $\Lambda^i(G^j_T,\alpha^i) > 0$ for all $\alpha^i > \alpha^i_{0T}$ at $G^j = G^j_T$. Furthermore, for $\alpha^i = \alpha^i_{0T}$, a marginal increase in G^j above G^j_T implies $\Lambda^i(G^j,\alpha^i) \gtreqless 0$ as $\Lambda^i_{G^j} \gtreqless 0$. When α^i rises marginally above α^i_{0T}, the value of G^j that restores the equality $\Lambda^i(G^j,\alpha^i) = 0$ will change according to $dG^j/d\alpha^i|_{\Lambda^i=0} = -\Lambda^i_\alpha/\Lambda^i_{G^j}$: $dG^j/d\alpha^i|_{\Lambda^i=0} \lesseqgtr 0$ iff $\Lambda^i_{G^j} \gtreqless 0$.

To see the implications of these observations, suppose that $\Lambda^i_{G^j} > 0$ in the neighborhood of G^j_T, which implies $\Lambda^i(G^j,\alpha^i_{0T}) > 0$ for G^j marginally above G^j_T, as illustrated in Fig. B.1.[3] In this case, a small increase in α^i above α^i_{0T} implies not only $\Lambda(\alpha^i,G^j_T) > 0$ at G^j_T, but also the value of G^j that restores the equality $\Lambda(\alpha^i,G^j) = 0$ falls. Thus, $B^i_T(G^j)$ shifts back from $B^i_A(G^j)$ to $\widetilde{B}^i_T(G^j)$ for values of G^j just below G^j_T given values of α^i just above α^i_{0T}, such that $B^i_T(G^j) = \widetilde{B}^i_T(G^j)$ over a larger range of G^j in the neighborhood of G^j_T. Alternatively, when $\Lambda^i_{G^j} < 0$ so that $\Lambda^i(G^j,\alpha_{0T}) > 0$ for G^j just below G^i_T, the marginal increase in α above α^i_{0T} implies that the point of discontinuity is above G^j_T. Then for values of G^j just above G^j_T, $B^i_T(G^j)$ shifts back from $B^i_A(G^j)$ to $\widetilde{B}^i_T(G^j)$. As such, the marginal

[2]For a given combination of $\alpha = \alpha^i_0$ and G^j that implies $\Lambda^i\left(G^j,\alpha\right) = 0$, it is possible that $\Lambda^i_{G^j} = 0$ for values of G^j in the neighborhood of that point of indifference, in which case country i would remain indifferent between trade and autarky in that neighborhood. Furthermore, it is possible that country i favors trade for all other possible values in G^j, or alternatively that it favors autarky for all other possible values of G^j. Although we cannot rule out such possibilities, we view them as highly unlikely, and focus on cases where $\Lambda^i_{G^j} \neq 0$.

[3]Thus, for G^j above the point of discontinuity (G^j_T), $B^i_T(G^j) = \widetilde{B}^i_T(G^j)$; and for $G^j < G^j_T$, $B^i_T(G^j) = B^i_A(G^j)$. In the case that $\Lambda^i_{G^i} < 0$, $B^i_T(G^j) = \widetilde{B}^i_T(G^j)$ for G^j below the discontinuity and $B^i_T(G^j) = B^i_A(G^j)$ for points above it.

increase in α^i implies once again that the range of G^js in the neighborhood of G_T^j for which $B_T^i(G^j) = \widetilde{B}_T^i\left(G^j\right)$ expands. Thus, the larger is the degree of comparative advantage α^i for country i relative to α_{0T}^i, the more likely it is that $\widetilde{B}_T^i(G_T^j) \in B_T^i(G_T^j)$. Since this is true for both i, (G_T^1, G_T^2) will be an equilibrium point under trade if $\alpha^i > \alpha_{0T}^i$ for both i.[4] ||

Proposition B.1 (*Equilibrium arming and the elasticity of substitution under complete symmetry.*) *Suppose the conditions of complete symmetry are satisfied and that labor and capital are sufficiently substitutable in the production of arms and/or the intermediate good. Furthermore, assume that each country's comparative advantage (α^i) is sufficiently strong. Then, an increase in the elasticity of substitution in consumption $\sigma > 1$ induces greater arming under trade (i.e., $dG_T/d\sigma > 0$). As σ approaches ∞, equilibrium arming under trade approaches equilibrium arming under autarky (i.e., $\lim_{\sigma\to\infty} G_T = G_A$).*

Proof: The implications of complete symmetry detailed in Section 5.1 together with $\sigma > 1$ imply that the FOC under trade (14) evaluated at at $G^i = G_T^* > 0$ can be written as

$$\frac{1}{m_T^i}\frac{\partial V_T^i}{\partial G^i} = \frac{1}{c}\left[\left(1-\frac{1}{\sigma}\right) rK_0\phi_{G^i}^i\big|_{G^i=G_T^*} - \left(1-\frac{1}{2\sigma}\right)\right]\psi = 0.$$

Factoring out $1 - 1/2\sigma$ from the RHS allows us to rewrite this condition as

$$\Omega^i(G_T^*, \sigma) = ArK_0\phi_{G^i}^i\big|_{G^i=G_T^*} - \psi = 0,$$

where now $A = (\sigma-1)/(\sigma-\frac{1}{2}) \in (0,1)$ and, by Proposition 2, $\Omega_G^i < 0$. A comparison of the expression above with the analogous condition under autarky in (12) after imposing the conditions of complete symmetry $G_A^i = G_A$ (i.e., $rK_0\phi_{G^i}^i\big|_{G^i=G_A} - \psi = 0$) reveals the conditions that guarantee the existence of a unique symmetric equilibrium in security policies under autarky ($G_A > 0$ for all $\sigma > 0$) also guarantee the existence of a unique, symmetric solution to $\Omega(G_T, \sigma) = 0$ for $\sigma > 1$. What's more, since $A < 1$, $G_T \in (0, G_A)$ for $\sigma \in (1, \infty)$. Our claim in the proposition that G_T is increasing in σ (for $\sigma > 1$) follows immediately since A is independent of of the common level of guns and is increasing in σ. To verify the limit part of the proof, observe that $\lim_{\sigma\to\infty} A = 1$. ||

Proof of Proposition 4 continued. The proof of part (b) of the proposition builds on the feature of the model that, as in the analysis with only two countries, the world price is bounded by the two autarky prices: $p_T \in [1/p_A^3, p_A]$. To keep matters simple but without loss of generality, we suppose that $a_1^1 = a_1^2 = a_2^3 = 1$ and $a_2^1 = a_2^2 = a_1^3 = \alpha > 1$, so that countries 1 and 2 have an identical comparative advantage in the production of good 1, whereas country 3 has a comparative advantage in the production of good 2; and,

[4] If $\alpha^i < \alpha_{0T}^i$, $G_T^i \neq \widetilde{G}_T^i$ and the equilibrium under free trade will involve mixed strategies and/or it will coincide with autarky.

furthermore, comparative advantage is symmetric across countries 1 and 2 on the one hand and country 3 on the other, with $\alpha > 1$ indicating the strength of that advantage. Given these simplifications, $p_A^i = \alpha$ for $i = 1, 2$ and $p_A^3 = \alpha$. To simplify a little more, suppose in addition that preferences are Cobb-Douglas ($\sigma = 1$) and identical across countries.[5] Then, the condition of balanced trade implies

$$p_T = \begin{cases} 1/\alpha & \text{if} \quad \pi \leq 1/\alpha \\ \pi & \text{if} \quad 1/\alpha < \pi < \alpha \\ \alpha & \text{if} \quad \alpha \leq \pi, \end{cases}$$

where $\pi \equiv \frac{\gamma_2}{\gamma_1}\left[\frac{Z^1+Z^2}{Z^3}\right]$. Since the expenditure shares ($\gamma_1 = 1 - \gamma_2$ for all three countries) are constant, the expression above shows that p_T remains fixed for combinations of Z^1 and Z^2 that imply (given Z^3) a constant sum $Z^1 + Z^2$.

We assume further (for clarity) that guns are produced with labor only (so that $\psi^i = w^i$), which implies that $Z^i = Z(L - G^i, K + \phi^i K_0)$, where $Z(\cdot)$ now represents a standard constant returns to scale production function that gives the output of the intermediate good Z^i; henceforth, we drop the "e" superscript on Z^i.[6] Then, the relevant upper bound on the world price can be written as

$$\sum\nolimits_{i=1,2} Z^i = \sum\nolimits_{i=1,2} Z(L - G^i, K + \phi^i K_0) = \frac{\gamma_1}{\gamma_2}\alpha Z^3 \equiv \overline{Z},$$

which defines combinations of guns such that $p_T = \alpha$.[7] This constraint is depicted in Fig. B.2, which also shows country 1's best-response function under autarky $B_A^1(G^2)$ and its best-response function under free trade that ignores the constraint on p_T, $\widetilde{B}_T^1(G^2)$.[8] Combinations of guns above the $p_T = \alpha$ constraint imply $p_T < \alpha$, whereas combinations on and below the constraint imply $p_T = \alpha$ (or equivalently $\sum_{i=1,2} Z^i \geq \overline{Z}$). Starting at points where the constraint binds, changes in guns by either country would cause p_T to fall only if the new point is beyond the $p_T = \alpha$ constraint.

The shape of this constraint, as shown in the figure, has three key properties:

(i) The constraint has a slope of -1 where it intersects the 45° line, as can be confirmed by evaluating $dG^2/dG^1|_{p_T=\alpha}$ at $G^2 = G^1$.

[5] The analysis does not change substantively if we allow $\sigma > 1$. In any case, note that we are not restricting preferences here to be symmetrically defined across the two goods.

[6] The assumption that guns are produced with labor alone is not crucial, but simplifies the analysis and guarantees uniqueness of equilibrium under autarky and free trade (assuming sufficiently large comparative advantage in the latter trade regime.

[7] A similar constraint can be written for the lower bound of $p_T = 1/\alpha$: $\sum_{i=1,2} Z^i = Z^3\gamma_1/(\gamma_2\alpha) \equiv \underline{Z}$, with $\underline{Z} < \overline{Z}$. However, it is the upper bound that is relevant here.

[8] That $B_A^1(G^2) < \widetilde{B}_T^1(G^2)$ for given G^2 reflects the effect of trade to augment a country's incentive to arm against another country when the two compete in the same export market, as emphasized in the text.

(ii) It is not possible for both G^1 and G^2 to increase as we move rightward along the constraint away from the 45° line towards $\widetilde{B}^1_T(G^2)$.[9]

(iii) The properties above in turn imply that the $p_T = \alpha$ constraint must intersect (or approach) $\widetilde{B}^1_T(G^2)$ at some point C' below and to the right of its intersection with the 45° line.[10]

Larger values of α imply that the $p_T = \alpha$ constraint lies closer to the origin, without affecting the positioning of $B^1_A(G^2)$ and $\widetilde{B}^1_T(G^2)$.[11]

Now consider the value of G^2 denoted by G^2_C such that $G^1_A = B^1_A(G^2_C)$ implies $\sum_{i=1,2} Z^i > \overline{Z}$ and, more importantly, such that $G^1_T = \widetilde{B}^1_T(G^2_C)$ implies $\sum_{i=1,2} Z^i = \overline{Z}$. For $G^2 = G^2_C$, a shift from autarky to free trade (shown in Fig. B.2 as a movement from point C to point C') implies $p_T = p_A = \alpha$. With no gains from trade given $G^2 = G^2_C$, country 1 prefers to stay on its best-response function under autarky that simply maximizes its output of Z^1. That is to say, $V^1_A(B^1_A(G^2_C), G^2_C) > \widetilde{V}^1_T(\widetilde{B}^1_T(G^2_C), G^2_C)$. Next consider a larger value of G^2 denoted by G^2_D such that $G^1_A = B^1_A(G^2_D)$ implies $\sum_{i=1,2} Z^i = \overline{Z}$ and furthermore $G^1_T = \widetilde{B}^1_T(G^2_D)$ implies $\sum_{i=1,2} Z^i < \overline{Z}$. In this case as shown in Fig. B.2, a shift from autarky to free trade implies $p_T < \alpha$ and induces country 1 to increase its arming in a move from $B_A(G^2_D)$ (point D) to $\widetilde{B}_T(G^2_D)$ (point D'), with $\widetilde{V}^1_T(\widetilde{B}^1_T(G^2_D), G^2_D) > V^1_A(B^1_A(G^2_D), G^2_D)$. By continuity, there exists a value of arming by country 2, denoted by $G^2_0 \in (G^2_C, G^2_D)$, such that $V^1_A(B^1_A(G^2_0), G^2_0) = \widetilde{V}^1_T(\widetilde{B}^1_T(G^2_0), G^2_0)$. This value of G^2_0 defines the discontinuity in country 1's best-response function under (free) trade, denoted by $B^1_T(G^2)$:

$$B^1_T(G^2) = \begin{cases} B^1_A(G^2) & \text{if } G^2 \le G^2_0; \\ \widetilde{B}^1_T(G^2) & \text{if } G^2 \ge G^2_0. \end{cases}$$

Since the location of the $p_T = \alpha$ constraint depends on α (moving closer to the origin as α increases), the location of the discontinuity depends on α as well.

We flesh out the possible welfare implications here, with the help of Fig. B.2. Point

[9] This property follows from the effects of changes in arming on $\sum_{i=1,2} Z^i$. Starting at the point where the $p_T = \alpha$ constraint intersects $B^1_A(G^2)$ in Fig. B.2, let G^1 rise while keeping G^2 fixed. The FOC under autarky (12) implies $\partial V^1_A/\partial G^1 < 0$ for $G^1 > B^1_A(G^2)$, so that $dZ^1/dG^1 < 0$; since $dZ^2/dG^1 < 0$ holds too, the increase in G^1 causes $\sum_{i=1,2} Z^i$ and thus p_T to fall, implying that the new combination of guns is above the $p_T = \alpha$ constraint. Repeated applications of this logic establish that G^1 and G^2 cannot both increase as we continue to move along the constraint approaching $\widetilde{B}^1_T(G^2)$). That is to say, the $p_T = \alpha$ constraint cannot be U-shaped to the right of the 45° line. However, it is possible that $dG^2/dG^1|_{p_T=\alpha} \to \infty$ somewhere along the constraint as we move further towards or beyond $\widetilde{B}^1_T(G^2)$—a possibility illustrated in Fig. B.2. There exist no combinations of G^1 and G^2 beyond this critical point, whether it is located to the left or right of $\widetilde{B}^1_T(G^2)$, where the price constraint binds.

[10] C' can be arbitrarily close to point 0, but that does not matter for our argument to follow.

[11] The $p_T = \alpha$ constraint likewise moves closer to the origin when either Z^3 or $\gamma_1 = 1 - \gamma_2$ increases, but such changes could also affect the positioning of $\widetilde{B}^1_T(G^2)$. Focusing on how the positioning of the $p_T = \alpha$ constraint depends on α allows us to show most clearly how the price constraint relates to the location of the discontinuity of the best-response function under free trade as derived below.

A on the 45° line where the best-response functions of the two (identical) countries under autarky would cross (so that $G_A^1 = G_A^2 = G_A$) represents the unique, symmetric equilibrium under autarky. Point T also on the 45° line shows where the unconstrained best-response functions under trade would cross so that $G_T^1 = G_T^2 = G_T$. Provided T lies above the $p_T = \alpha$ constraint, it represents an equilibrium under free trade. A movement along the 45° line from A to T implies no change in the distribution of K_0, but higher security costs. How these added security costs compare with the gains from trade depends on the location of the discontinuity of the constrained best-response function under trade, $B_T^1(G^2)$.

Suppose that the discontinuity occurs at $G_0^2 = G_A$, depicted as point A. By the definition of the discontinuity, the payoffs to country 1 under autarky and trade will be equal at this level of arming by country 2: $V_A^1(B_A^1(G_A), G_A) = \widetilde{V}_T^1(\widetilde{B}_T^1(G_A), G_A)$. We now show that a movement along $\widetilde{B}_T^1(G^2)$ from $(\widetilde{B}_T^1(G_A), G_A)$ in the direction of point T reduces country 1's payoff in the 3-country case. By the envelope theorem, since we are moving along country 1's best-response function under trade, we need only to consider the welfare effects of a change in G^2 as it influences country 1's optimal production of the intermediate input and p_T. Keeping in mind the effects a change in G^2 on both countries' optimizing production, dZ^1/dG^2 and dZ^2/dG^2 shown in (11), we use (16) with $\Delta = \sigma = 1$ and rearrange to find:

$$\frac{1}{V_T^1}\frac{\partial V_T^1}{\partial G^2} = \left[1 - \nu^1\gamma_2\right]\frac{dZ^1/dG^2}{Z^1} - [\nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2}. \tag{B.9}$$

To evaluate the sign of (B.9), observe that, for points on $B_T^1\left(G^2\right)$ where $G^2 < B_T^2\left(G^1\right)$, the net marginal benefit of arming for country 2 is positive:

$$\frac{1}{V_T^2}\frac{\partial V_T^2}{\partial G^2} = [1 - \nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2} - [\nu^1\gamma_2]\frac{dZ^1/dG^2}{Z^1} > 0.$$

This inequality can be rewritten as

$$-[\nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2} < -\left[\frac{\nu^1\nu^2\left(\gamma_2\right)^2}{1-\nu^2\gamma_2}\right]\frac{dZ^1/dG^2}{Z^1},$$

and then combined with (B.9) (using the fact that $\nu^1 + \nu^2 = 1$) to find

$$\frac{1}{V_T^1}\frac{\partial V_T^1}{\partial G^2} < \left[\frac{1-\gamma_2}{1-\nu^2\gamma_2}\right]\frac{dZ^1/dG^2}{Z^1}.$$

Since $\nu_2, \gamma_2 < 1$ and $dZ^1/dG^2 < 0$, the RHS of the expression above is negative, which in turn implies that $\partial V_T^1/\partial G^2 < 0$.[12] As such, $V_T(G_T, G_T) < V_A(G_A, G_A)$ when $G^2 = G_A$.

By continuity, there exist higher values of α (with the $p_T = \alpha$ constraint and the

[12] A sufficient (but not necessary) condition for this result to hold more generally under CES preferences is that $\sigma \geq 1$.

discontinuity in $B_F^1(G^2)$ moving towards the origin), such that the gains from trade continue to be less than the higher security costs under trade, implying that a shift from autarky to free trade is welfare reducing. Of course, increases in the strength of comparative advantage eventually imply sufficiently large gains from trade that swamp the higher security costs and thus render free-trade Pareto preferred to autarky.[13] $\|$

B.2 Trade Costs

Trade in world markets can be costly first due to the possible existence of geographic or physical trade barriers that take Samuelson's "iceberg" form. In particular, for each unit of a final good that country i imports (and consumes), country j must ship τ^i (≥ 1) units. Trade across national borders might be costly also due to the existence of import tariffs. Denote country i's (ad valorem) import tariff plus 1 by t^i (≥ 1).[14]

B.2.1 Introducing Trade Costs

Our analysis of trade must be modified in in two principal ways to account for trade costs. First, we need to distinguish between domestic and world prices. Specifically, letting q_j denote the "world" price of good j, competitive pricing implies $q_j = p_j^j = c^j$ for $j = 1, 2$ and $q^i = q_j/q_i = c^j/c^i$. But, the presence of trade costs drives a wedge between this world relative price and the domestic relative price of that same good imported by country i, $p^i = p_j^i/p_i^i$. In particular, $p_T^i = t^i \tau^i q^i$, which implies $\widehat{p}_T^i = \widehat{t}^i + \widehat{\tau}^i + \widehat{q}_T^i$. Hereafter, we omit the subscript "T" to avoid clutter.

Second, we must capture the presence of tariff revenues.[15] Assume that each country i specializes completely in producing the good i, so that its demand for good j is satisfied entirely through imports: $M^i = D_j^i$, where from (6) $D_j^i = \gamma_j^i Y^i / p_j^i$. Then,

$$Y^i = c^i Z^i + \tfrac{t^i - 1}{t^i} p_j^i D_j^i = c^i Z^i + \tfrac{t^i - 1}{t^i} \gamma_j^i Y^i \quad \Longrightarrow \quad Y^i = \frac{c^i Z^i}{\gamma_i^i + \gamma_j^i / t^i}, \quad i \neq j. \tag{B.10}$$

Substitution of the second expression for Y^i in (B.10) back into D_j^i gives

$$D_j^i = \frac{\gamma_j^i Z^i / p^i}{\gamma_i^i + \gamma_j^i / t^i}, \quad i \neq j = 1, 2. \tag{B.11}$$

The (absolute value of the) uncompensated (Marshallian) price elasticity of import demand

[13]Note that for smaller α, the discontinuity in $B_T^1(G^2)$ moves between points A and T. In such cases, both points A and T represent pure-strategy equilibria, with autarky being Pareto preferred to free trade. For α sufficiently small to push the $p_T = \alpha$ constraint beyond point T, the only pure-strategy equilibrium is autarky.

[14]We abstract from the possible existence of internal transportation costs and export taxes.

[15]We assume these revenues are distributed to consumers in a lump sum fashion.

ε^i and the (absolute value of the) compensated (Hicksian) price elasticity of import demand η^i can be computed respectively as follows:

$$\varepsilon^i \equiv -\frac{\partial M^i/\partial p_T^i}{M^i/p_T^i} = 1 + (\sigma - 1)\frac{t^i\gamma_i^i}{t^i\gamma_i^i + \gamma_j^i} \tag{B.12a}$$

$$\eta^i \equiv -\left.\frac{\partial M^i/\partial p_T^i}{M^i/p_T^i}\right|_{dU^i=0} = \varepsilon^i - \frac{\gamma_j^i}{t^i\gamma_i^i + \gamma_j^i} = \sigma\left(\frac{t^i\gamma_i^i}{t^i\gamma_i^i + \gamma_j^i}\right) > 0, \tag{B.12b}$$

for $i \neq j = 1, 2$. Note that $\text{sign}\left\{\varepsilon^i - 1\right\} = \text{sign}\left\{\sigma - 1\right\}$.

Given countries specialize completely in the production of their respective exported goods, world market clearing requires $\tau^i q_j D_j^i = \tau^j q_i D_i^j$. Using (B.11) and the pricing relations spelled out above, one can verify this condition implies:

$$q^i\left(\frac{\gamma_j^i Z^i}{t^i\gamma_i^i + \gamma_j^i}\right) = \frac{\gamma_i^j Z^j}{t^j\gamma_j^j + \gamma_i^j},\ i \neq j = 1, 2.$$

Thus, the world market-clearing price q^i is implicitly defined as a function of security policies and trade costs. In what follows, we assume this condition is satisfied for $p^i = \tau^i t^i q^i \in [1/\alpha^j, \alpha^i]$ for $i \neq j = 1, 2$, so as to abstract from the possible issues that can arise in relation to discontinuities in the best-response functions. More to the point, assuming a sufficiently large comparative advantage (α^i) for each country i, given trade costs, allows us to focus on the salient features of the trade equilibrium that involves strictly positive trade flows and thus is distinct from the equilibrium under autarky.

Totally differentiating this expression and rearranging terms allows us to trace out the effects of Z^i, Z^j, t^i, t^j, τ^i and τ^j on the market-clearing relative world price of country i's imported good, q^i:

$$\widehat{q}^i = \frac{1}{\Delta}\left[\widehat{Z}^i - \widehat{Z}^j - \eta^i\widehat{t}^i + \eta^j\widehat{t}^j - \left(\varepsilon^i - 1\right)\widehat{\tau}^i + \left(\varepsilon^j - 1\right)\widehat{\tau}^j\right], \tag{B.13}$$

where η^i and ε^i are shown in (B.12) and where we assume the Marshall-Lerner condition for stability, given by $\Delta \equiv \varepsilon^i + \varepsilon^j - 1 > 0$, is satisfied. As expected, an increase (decrease) in t^i (t^j) improves country i's terms of trade. Furthermore, if $\sigma > 1$, then a change in τ^i (τ^j) has a qualitatively similar effect on q^i. The effects of changes in Z^i and Z^j on q^i are as expected. If the percentage increase in Z^i (brought about by a change in, say, G^i) exceeds the corresponding change in Z^j, then i's terms of trade will deteriorate.

B.2.2 Payoff Functions and the Incentive To Arm

The payoff functions under costly trade are central to characterizing the countries' optimizing choices of guns production. Recall we can write $V^i = \mu^i Y^i$, where μ^i is the inverse of the price index, given by $P^i = p_i^i[1 + (p^i)^{1-\sigma}]^{1/(1-\sigma)}$. Then, using the expression for

$Y^i = c^i Z^i = p_i^i Z^i$ shown in (B.10), we can rewrite the indirect utility function as

$$V^i = \frac{p_i^i Z^i / P^i}{\gamma_i^i + \gamma_j^i / t^i}, \quad i \neq j = 1, 2. \tag{B.14}$$

Total differentiation of (B.14), with a focus on percentage changes, yields

$$\widehat{V}^i = \widehat{Z}^i + \frac{\gamma_j^i}{t^i \gamma_i^i + \gamma_j^i} \widehat{t}^i - \frac{\gamma_j^i}{t^i} \left[(t^i - 1)\varepsilon^i + 1 \right] \left(\widehat{t}^i + \widehat{\tau}^i + \widehat{q}^i \right). \tag{B.15}$$

Upon substituting the expression for $\widehat{q}_T^i$ (B.13) into (B.15) and simplifying, one can verify

$$\begin{aligned} \widehat{V}^i \quad = \quad & \left(1 - \rho^i\right) \widehat{Z}^i + \rho^i \widehat{Z}^j - \rho^i \left[\varepsilon^j \widehat{\tau}^i + \left(\varepsilon^j - 1\right) \widehat{\tau}^j + \eta^j \widehat{t}^j \right] \\ & + \left(\gamma_j^i / t^i \Delta\right) \eta^i \left[1 - \left(t^i - 1\right) \left(\varepsilon^j - 1\right) \right] \widehat{t}^i, \end{aligned} \tag{B.16}$$

for $j \neq i = 1, 2$, where $\rho^i \equiv (\gamma_j^i / t^i \Delta)[\left(t^i - 1\right) \varepsilon^i + 1]$. The first two terms in the first line identify the effects of changes in Z^i and Z^j respectively. The next term in the first line identifies the effects of changes in geographic trade costs for countries i and j and tariffs imposed by country j. The term on the second line shows the effect of changes in tariffs imposed by the country i. Observe especially that this formulation allows for asymmetric trade costs across the two countries.

To explore how the presence of trade costs matter for arming incentives, we now consider the effect of an increase in country i's own guns G^i on its payoff. For example, if there are non-tariff barriers ($\tau^i > 1$) but no tariffs ($t^i = 1$ so that $\rho^i = \gamma_j^i / \Delta$), then (B.16) implies the following net marginal benefit from arming for country i:

$$\left. \frac{\partial V_T^i / \partial G^i}{V_T^i} \right|_{t^i = 1} = \left(1 - \frac{\gamma_j^i}{\Delta}\right) \frac{dZ^i / dG^i}{Z^i} + \left(\frac{\gamma_j^i}{\Delta}\right) \frac{dZ^j / dG^i}{Z^j}. \tag{B.17}$$

Observe that this expression is similar to what we have under free trade, shown in (13), with the only difference being the replacement of Δ for σ.[16] Alternatively, suppose each country simultaneously chooses its tariff and security policies to maximize its payoff. From (B.16), a necessary condition for i's tariff to be optimal (given $\sigma > 1$) is that $t^i - 1 = 1/(\varepsilon^j - 1)$ for $i \neq j = 1, 2$ or, equivalently, $t_o^i = \varepsilon^j / (\varepsilon^j - 1)$, which is the standard optimal tariff formula. From the definition of $\Delta \equiv \varepsilon^i + \varepsilon^j - 1 > 0$, we have $\rho^i = \gamma_j^i / \varepsilon^j$; and, from (B.16), country i's net marginal benefit from arming in percentage terms becomes:

$$\left. \frac{\partial V^i / \partial G^i}{V^i} \right|_{t^i = t_o^i} = \left(1 - \frac{\gamma_j^i}{\varepsilon^j}\right) \frac{dZ^i / dG^i}{Z^i} + \left(\frac{\gamma_j^i}{\varepsilon^j}\right) \frac{dZ^j / dG^i}{Z^j}. \tag{B.18}$$

[16] As one can easily verify, $\Delta = \sigma$ when $\tau^i = t^i = 1$.

Now recall, from (11b), that an increase in G^i given G^j implies less of the contested resource for country j—i.e., $dZ^j/dG^i < 0$ holds. Furthermore, while country i recognizes this effect in its choice of arms under trade, it ignores this effect under autarky. Instead, as established earlier, each country i under autarky chooses G^i effectively to maximize the income from its production of Z^i, which implies $dZ^i/dG^i = 0$. Accordingly, for each of the two cases above as well as more generally, country i's net marginal benefit from arming under trade evaluated at G_A^i given G^j is negative.[17] Hence, country i's incentive to arm (given G^j) is smaller under trade, whether costly or not. The only restriction we make here is that, given trade costs, the strength of comparative advantage is sufficiently large to allow for strictly positive trade flows.

B.2.3 Numerical Results

As was the case under free trade, a shift from an equilibrium under autarky to one with trade could induce one adversary to increase its arming. Because the equilibrium under (costly) trade is intractable analytically, we establish this point with the help of numerical methods. In addition, we extend the analysis to explore the dependence of arming on iceberg-type transportation costs and tariffs. Finally we compare the various equilibria to the ones that would arise if countries chose their tariffs and security policies optimally.

To identify the effects of changes in trade costs on equilibrium arming choices and payoffs, let $V^i_{G^i} = 0$ $(i = 1, 2)$ be the FOC condition associated with country i's arming under trade. (Again, when feasible, we omit subscript "T" to avoid clutter.) Now let s denote a geographic or policy related trade cost. We then ask what is the impact of an increase in s on the two countries' arming decisions. It is straightforward to show

$$\frac{dG^i}{ds} = \frac{1}{D}\left[\overset{(?)}{V^i_{G^iG^j}}\overset{(?)}{V^j_{G^js}} - \overset{(-)}{V^j_{G^jG^j}}\overset{(?)}{V^i_{G^is}}\right] \tag{B.19a}$$

$$\frac{dG^j}{ds} = \frac{1}{D}\left[-\overset{(-)}{V^i_{G^iG^i}}\overset{(?)}{V^j_{G^js}} + \overset{(?)}{V^j_{G^jG^i}}\overset{(?)}{V^i_{G^is}}\right], \tag{B.19b}$$

where $D \equiv V^i_{G^iG^i}V^j_{G^jG^j} - V^i_{G^iG^j}V^j_{G^jG^i}$. To ensure the equilibrium in arming is stable, we assume $D > 0$. We also assume $V^i_{G^iG^i} < 0$ and $V^j_{G^jG^j} < 0$, so that the second-order condition for each country's optimizing choice of guns is satisfied. Clearly, the impact of a change in s on arming depends on (i) the sign of its direct effect on both countries' marginal payoffs due to arming and, (ii) on whether their guns are strategic substitutes or strategic complements.

Our numerical simulations are based on a particular parameterization of the model that assumes guns are produced by both countries with labor only and on a one-to-one basis,

[17] It is also possible to show, in a setting without tariffs, a decrease in either country i's non-tariff trade barriers (i.e., where initially $\tau^i > 1$) reduces each country's net marginal benefit from arming.

implying $Z^i = \left(K^i + \phi^i K_0\right)^{1-\theta}\left(L^i - G^i\right)^{\theta}$, where $\theta \in (0,1)$. When the distribution of secure resources is sufficiently even to imply that both countries' security policies exhibit strategic complementarity in the neighborhood of the autarkic equilibrium, the analysis yields predictable results. Thus, while we consider such cases below, we focus largely on those cases where the distribution is extremely uneven, using figures to illustrate. The figures assume $\theta = 0.2$, $\alpha = \infty$, $K^1 = L^1 = 1.95$ and $K^2 = L^2 = 0.05$, with $K_0 = 1$.[18]

Geographic trade costs. Let us consider $s = \tau^i$, and assume neither country imposes tariffs: $t^i = t^j = 1$. Then, country i's FOC with respect to arming at an interior solution requires $\frac{\partial V^i/\partial G^i}{V^i}|_{t^i=1} = 0$. It is straightforward to show, using (B.17), that $V^i_{G^i\tau^i} > 0$ and $V^j_{G^j\tau^i} > 0$.[19] Accordingly, the following ideas can also be established with the help of numerical analysis.

(a) If the countries' secure factor endowments are sufficiently similar such that their security policies are strategic complements, then $dG^i/d\tau^i > 0$ and $dG^j/d\tau^i > 0$ hold. It also follows that $dG^i/d\tau > 0$ and $dG^j/d\tau > 0$ for $\tau^i = \tau^j = \tau$.

(b) Now suppose the international distribution of secure factor endowments is uneven, with country i having sufficiently greater secure endowments of both capital and labor to imply $V^i_{G^iG^j} > 0$ while $V^j_{G^jG^i} < 0$. Based on the preceding analysis, it follows that $dG^i/d\tau^i > 0$, as illustrated for country $i = 1$ in Fig. B.3(a). Furthermore, numerical simulations indicate that $dG^j/d\tau^i > 0$ under most situations, which include the possible presence of asymmetric endowments. Nevertheless, for sufficiently uneven distributions of secure endowments across countries, $dG^j/d\tau^i < 0$ can hold, as illustrated in Fig. B.3(b) for $j = 2$. This figure also illustrates that G^j responds non-monotonically to changes in trade costs τ^i. Moreover, it is possible for $G^j_T > G^j_A$ to hold at some trade cost levels, including trade costs that are sufficiently close to 1 (free trade), as discussed earlier in the paper. This possibility can be seen in the figure by comparing the curve labeled as $\tau^2 = \infty$ (which corresponds to autarky, since $\alpha = \infty$) with the other ones.

(c) Nonetheless, under most circumstances, globalization (i.e., $\tau^i \downarrow$ and/or $\tau^j \downarrow$) induces both adversaries to reduce their equilibrium arming. Exhaustive numerical analysis also establishes that $d(G^i_T + G^j_T)/d\tau^i > 0$ under all circumstances (even when $dG^j/d\tau^i < 0$, as expected since country j's sufficiently small size implies this effect is relatively small).

[18] These assumptions imply that the two countries initially hold an identical ratio of capital to labor. However, what matters in determining whether one country's best-response function in the neighborhood of the autarky equilibrium depends positively or negatively on its rival's arming is the similarity of their ratios of *residual* secure capital to *residual* secure labor, k^i_X shown in (4), after arming choices have been made.

[19] That is, $\text{sign}\{V^i_{G^i\tau^i}\} = -\,\text{sign}\{d\rho^i/d\tau^i\} = -\,\text{sign}\{d(\gamma^i/\Delta)/d\tau^i\} > 0$ and similarly for the sign of $V^j_{G^j\tau^i}$.

Turning to payoffs, inspection of the welfare decomposition shown in (B.16) reveals that the direct effect of globalization on either country's payoff is positive. Since the smaller country's rival reduces its arming, the strategic payoff effect reinforces that direct effect. Hence, globalization is always welfare enhancing for smaller countries. For the larger country, whose rival arms by more as with globalization, the strategic effect moves in the opposite direction. However, exhaustive numerical analysis confirms that, even in this case, the direct (and positive) effect dominates. Thus, we have $dV_T^i/d\tau^i < 0$ and $dV_T^j/d\tau^i < 0$, which also suggests that $V_T^i > V_A^i$ for $i = 1, 2$.

Tariffs. Now we consider $s = t^i$, assuming no geographic trade costs: $\tau^i = \tau^j = 1$. In this case, country i's FOC with respect to arming at an interior solution requires $\frac{\partial V^i/\partial G^i}{V^i}|_{\tau^i=1} = 0$. Analyzing the effects of tariffs on equilibrium arming and payoffs is considerably more complex than analyzing the effects of geographic trade barriers. Specifically, because tariffs generate revenues, signing the direct effects of tariffs on the marginal payoffs to arming $V_{G^it^i}^i$ and $V_{G^jt^i}^j$ becomes very difficult.[20] Matters are further complicated by the dependence of the parameter ρ^i on the signs of the cross partials $V_{G^iG^j}^i$, which are themselves difficult to identify since changes in security policies also affect prices. Finally, establishing quasi-concavity of V^i in G^i and uniqueness of equilibrium under trade is also difficult. Even so, we can use numerical methods to calculate (the unique) equilibrium under trade and its dependence on tariffs. Highlighting our more interesting findings, we have:

(a) When the distribution of secure resources is severely uneven, the larger country i's security policy G^i can be non-monotonic in its own tariff t^i. In particular, as illustrated in Fig. B.4(a) for $i = 1$, whether $t^2 = 1$, $t^2 = 2$, or $t^2 = t^1$, $dG_T^1/dt^2 < 0$ at low t^1 levels, but $dG_T^1/dt^1 > 0$ at high t^1 levels.[21] This finding suggests that a large country's (i) trade policy (t^i) can serve as a substitute for its security policy (G^i) at low tariff levels but as a complement at high tariff levels. By contrast, $dG_T^j/dt^i > 0$ at low values of t^i, whereas $dG_T^j/dt^i < 0$ at high t^i values, as illustrated in Fig. B.4(b) for $j = 2$. Thus, a small country (j) responds to protectionist moves by its relatively large adversary (i) by increasing (reducing) its arming G^j at low (high) tariff rates.

(b) The findings above do change with alternative endowment configurations. In the case of complete symmetry, for example, numerical analysis suggests the emergence non-monotonicities with respect to changes in t^i depends on the value of t^j. In particular,

[20] From (B.16), we see that $\text{sign}\{V_{G^it^i}^i\} = -\ \text{sign}\{d\rho^i/dt^i\}$, where ρ^i now depends on t^i directly and indirectly through its dependence on expenditure shares and uncompensated price elasticities of import demands which, in turn, depend on t^i directly and indirectly through internal and external prices. The sign of $V_{G^jt^i}^j$ is simpler to compute since it does not depend directly on t^i. However, it, too, depends on the various elasticities noted above and expenditure shares.

[21] The last case where $t^2 = t^1$ is relevant when countries agree to sign reciprocal trade agreements that involve equal tariff concessions to each other.

when t^j is not large, more liberal trade policies by country i ($t^i \downarrow$) are accompanied by less aggressive security policies by both countries; $dG_T^i/dt^i > 0$ and $dG_T^j/dt^i > 0$.

(c) Numerical analysis reveals further (not shown) that trade liberalization by a small county i ($t^i \downarrow$) always induces a less aggressive security policy by its larger rival (j): $dG^j/dt^i > 0$. But, such a trade policy by country i tends to affect its own security policy non-monotonically and in a way that depends on its rival's (j) trade policy. For example, if $t^j = 1$, then $\lim_{t^i \to 1} dG^i/dt^i > 0$, but $dG^i/dt^i < 0$ at large t^i values; however, if t^j is large, then $dG^i/dt^i < 0$ for all t^i.

Our numerical analysis also reveals a number of interesting tendencies for equilibrium arming under various non-cooperative equilibria. Specifically, we compare arming under (i) autarky ($t^j = \infty$), (ii) free trade ($t^i = t^j = 1$) and (iii) "generalized war" ($t^i = t_N^i$) which we identify with the Nash equilibrium in tariff and security policies).[22] Letting the subscripts "A," "F," and "W" respectively denote equilibrium values under autarky, free trade and generalized war, we observe the following tendences:

(a) Under complete symmetry (and most circumstances):
$G_F^i < G_W^i < G_A^i$ for $i = 1, 2$.

(b) When country i (j) is extremely large (small):
$G_W^i < G_F^i < G_A^i$, while $G_A^j < G_F^j < G_W^j$ for $i \neq j$.

As expected, under complete symmetry (and more generally), autarky induces both countries to produce more guns as compared with the other equilibria. Free trade leads to relatively less arming by each. However, for extremely uneven distributions in factor endowments, matters change, as illustrated in Figs. B.4(a) and B.4(b).[23] While the large country ($i = 1$) continues to produce more arms under autarky, it tends to arm by less under a generalized war than under free trade. We view this latter result as reflecting the added flexibility (and muscle) afforded by generalized war for this country to manipulate its terms of trade more effectively via its trade policy t^1. The relatively smaller country ($j = 2$) tends to arm by more under a generalized war than under free trade for exactly the opposite reasons. The comparison of aggregate arming across these regimes is determined by the relatively larger economy's guns.

We now consider the welfare implications of tariffs using (B.16). Focusing on dV_T^i/dt^i, first, note that the direct effect of an increase in t^i on V_T^i is positive (negative) if $t^i < t_o^i$

[22] The necessary FOCs for an interior equilibrium of the simultaneous-move game, which to the best of our knowledge has not been studied before, are: $V_{G^i}^i = 0$ and $V_{t^i}^i = 0$ ($i = 1, 2$), which involve four equations in four unknowns. $V_{G^i}^i = 0$ is familiar from our earlier analysis. $V_{t^i}^i = 0$ requires each country i to use its tariff to balance at the margin its welfare gain due to a terms-of-trade improvement against its welfare loss due to an adverse volume-of-trade effect. As noted earlier, $t^i = t_o^i$ where $t_o^i = \varepsilon^j/(\varepsilon^j - 1)$, $j \neq i = 1, 2$.

[23] That the quantities of guns associated with these equilibria are shown as being independent of t^1 in the figures is due to the fact that both guns and tariffs are endogenous.

($t^i > t^i_o$). We also need to examine the indirect effects. But, by the envelope theorem, only the effect of t^i on G^j is relevant. Keeping in mind that $\partial V^i_T/\partial G^j < 0$, the nature of this strategic effect depends on the initial level of t^i. Fig. B.5(a) shows, for $i = 1$, that $\lim_{t^i\to 1} \partial V^i_T/\partial t^i > 0$ and $\lim_{t^i\to\infty} \partial V^i_T/\partial t^i < 0$. The same figure also illustrates that it is possible for country 1 to use its trade policy to improve its payoff beyond the level under free trade V^1_F.

Turning to the effects of t^i on V^j_T, we can infer from (B.16) that the direct effect is negative and that indirect effect due a change in G^i_T depends on the initial level of t^i. In the case depicted in Figs. B.4 and B.5 for country $j = 2$, the indirect effect is positive for small t^i values and negative for large t^i values. Still, it appears that the direct effect dominates, so that $dV^2_T/dt^1 < 0$. Because in all other cases the strategic effect is negative, we also have $dV^2_T/dt^1 < 0$.

Finally, we compare the payoffs under the various equilibria identified above (i.e., autarky, generalized war, and free trade) and also throw into the mix the payoffs often considered in the literature that presumes no resource insecurity and thus no arming. Specifically, we consider free-trade under "Nirvana" (or no arms) indicated with the subscript "FN" and a trade war under "Nirvana" (again no arms) indicated with the subscript "WN." The payoff rankings depend on the distribution of secure endowments as follows:

(a) Under complete symmetry (and most circumstances):
$V^i_A < V^i_W < V^i_F < V^i_{WN} < V^i_{FN}$ for $i = 1, 2$.

(b) When country i (j) is extremely large (small):
$V^i_{FN} < V^i_{WN} < V^i_A < V^i_F < V^i_W$ and $V^j_A < V^j_W < V^j_F < V^j_{WN} < V^j_{FN}$ for $i \neq j$.

As is well-known from standard theory, under complete symmetry and secure property, a trade war (WN) has the features of a prisoner's dilemma problem relative to free trade (FN), such that $V^i_{WN} < V^i_{FN}$. What is not known is that we also have $V^i_A < V^i_W < V^i_F$ in this case. Interestingly, this ranking of payoffs is preserved for the smaller country when the distribution of secure endowments is sharply uneven. By contrast, the payoff rankings for larger country change in this case. We see that it prefers a generalized war (W) over a non-cooperative equilibrium in security policies coupled with free trade (F). While this result might seem surprising, it is consistent with the finding of Syropoulos (2002) that, in the absence of insecure property, a sufficiently large country "wins" a tariff war over its smaller trading partner. But, what is perhaps striking is that the extremely large country prefers all of these equilibria to the ones under Nirvana (WN and FN).

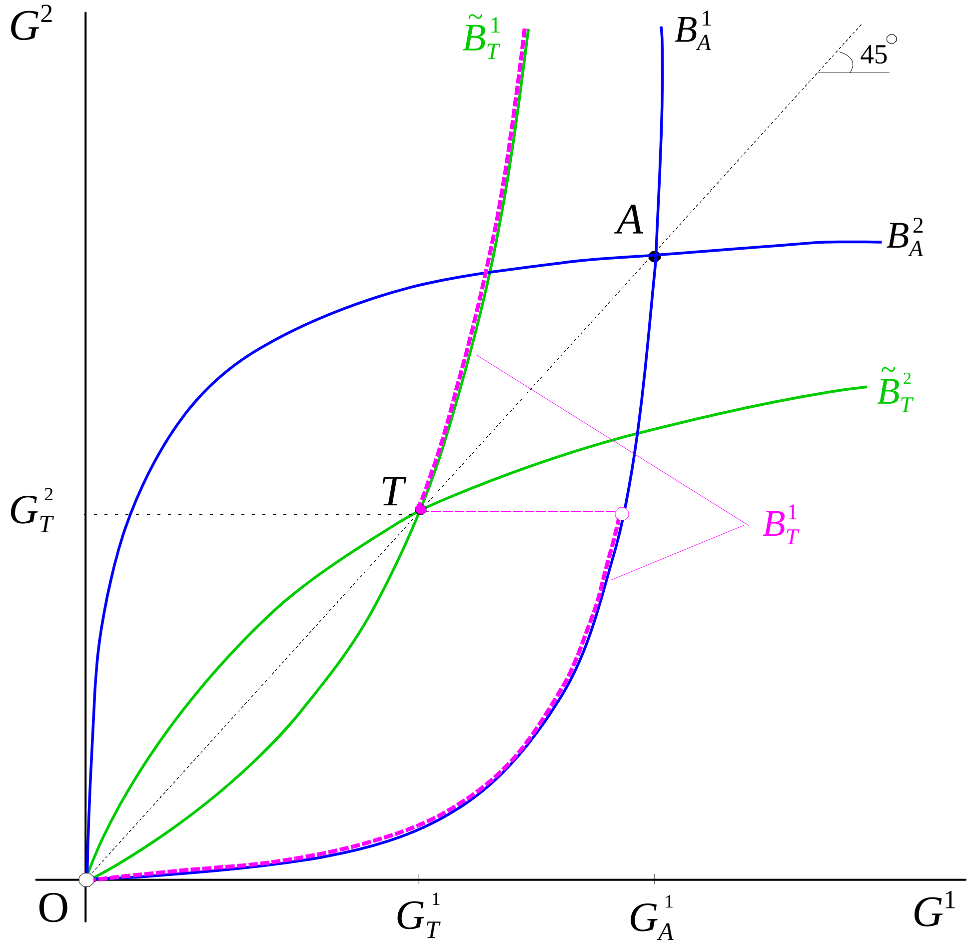


Figure B.1: Derivation of Best-Response Function under Trade and Nash Equilibria in Security Policies in the Two-Country Case

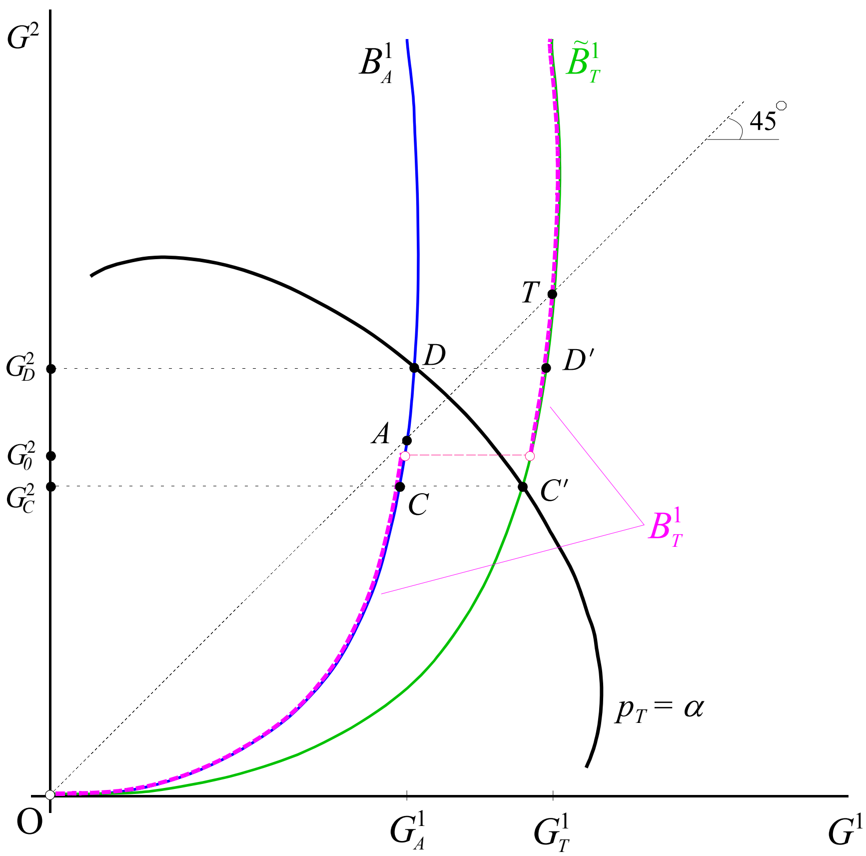


Figure B.2: Derivation of Best-Response Function under Free Trade and Comparison of Nash Equilibria in Security Policies in the Three-Country Case

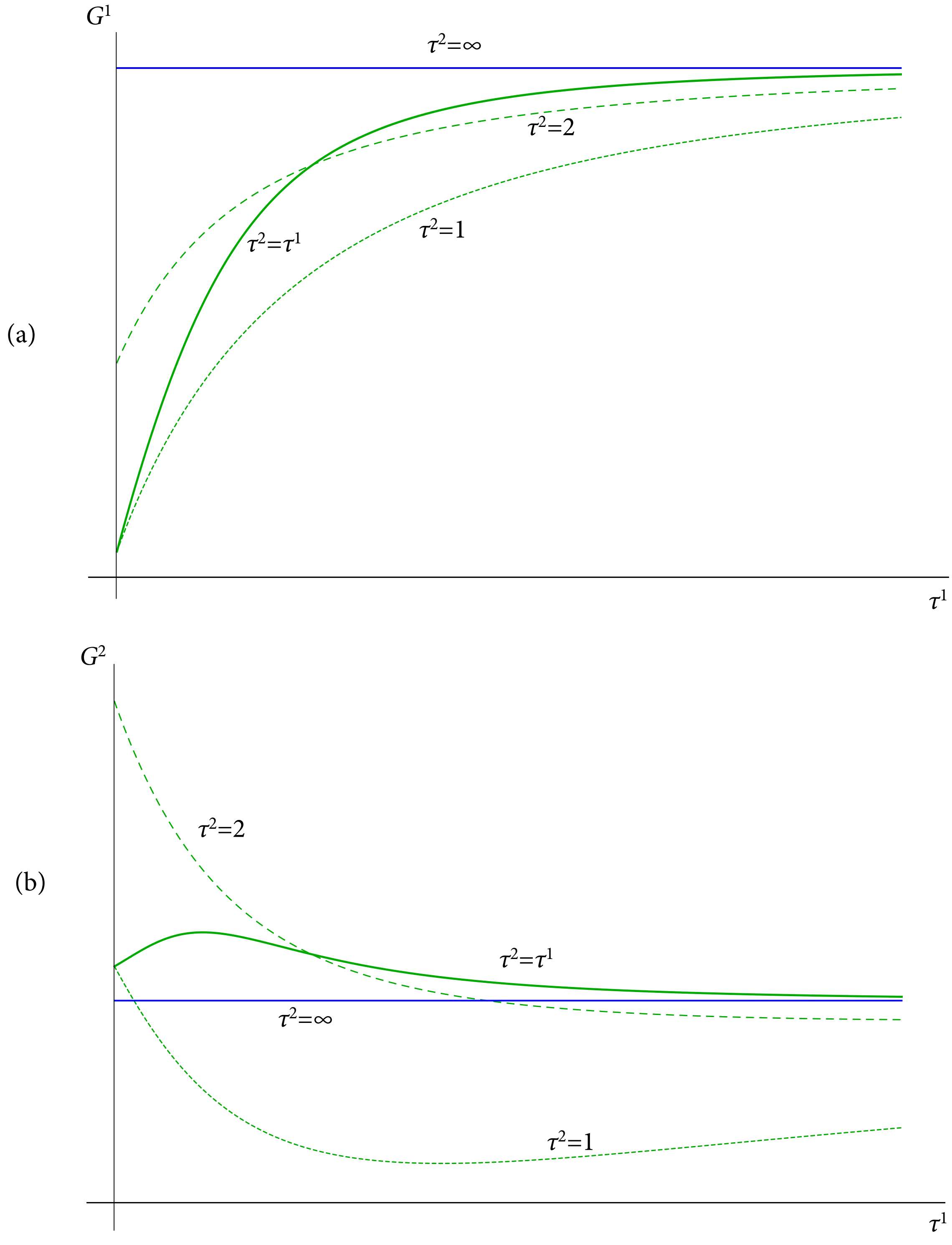


Figure B.3: The Dependence of Arming on Geographic Trade Costs

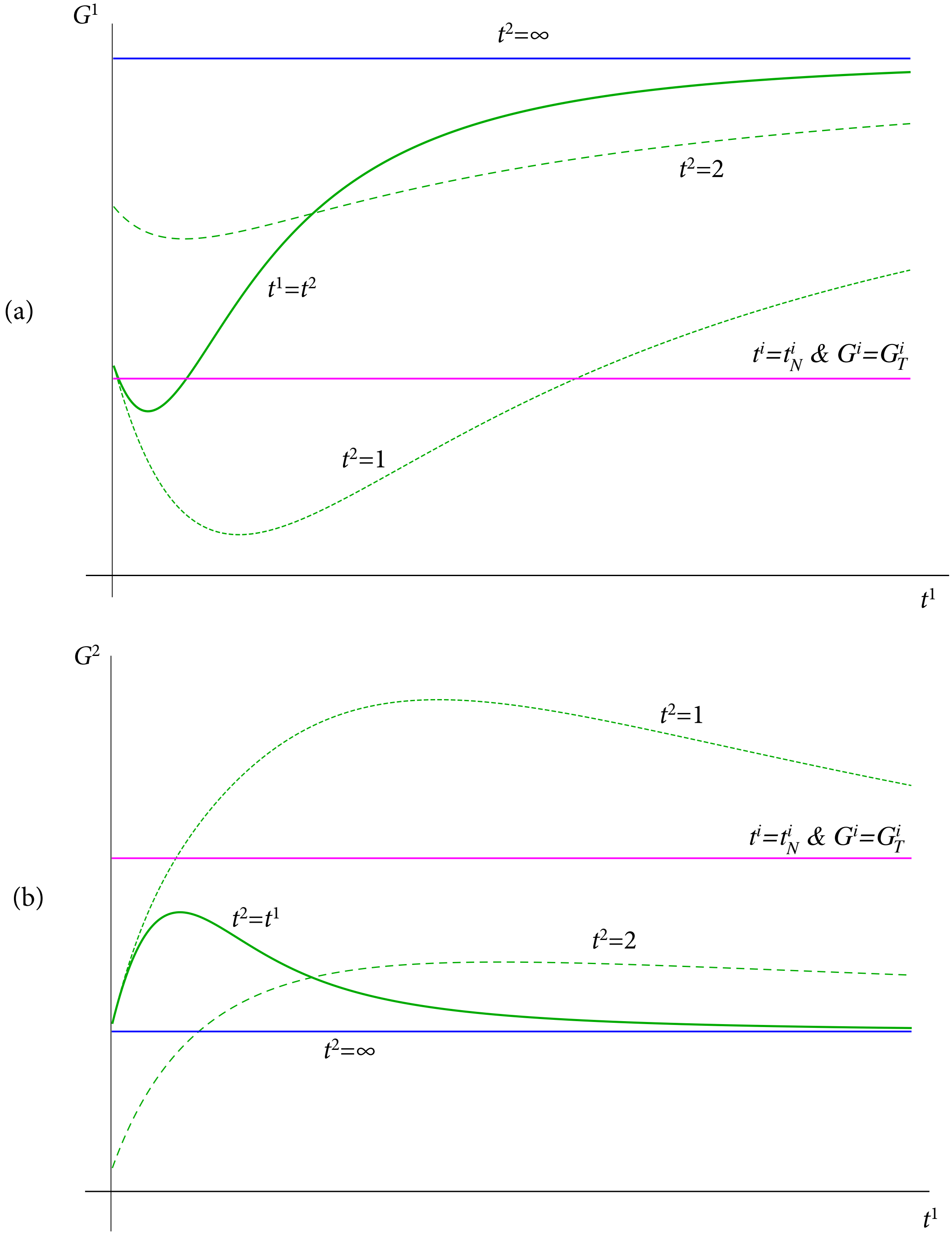


Figure B.4: The Dependence of Arming on Tariffs

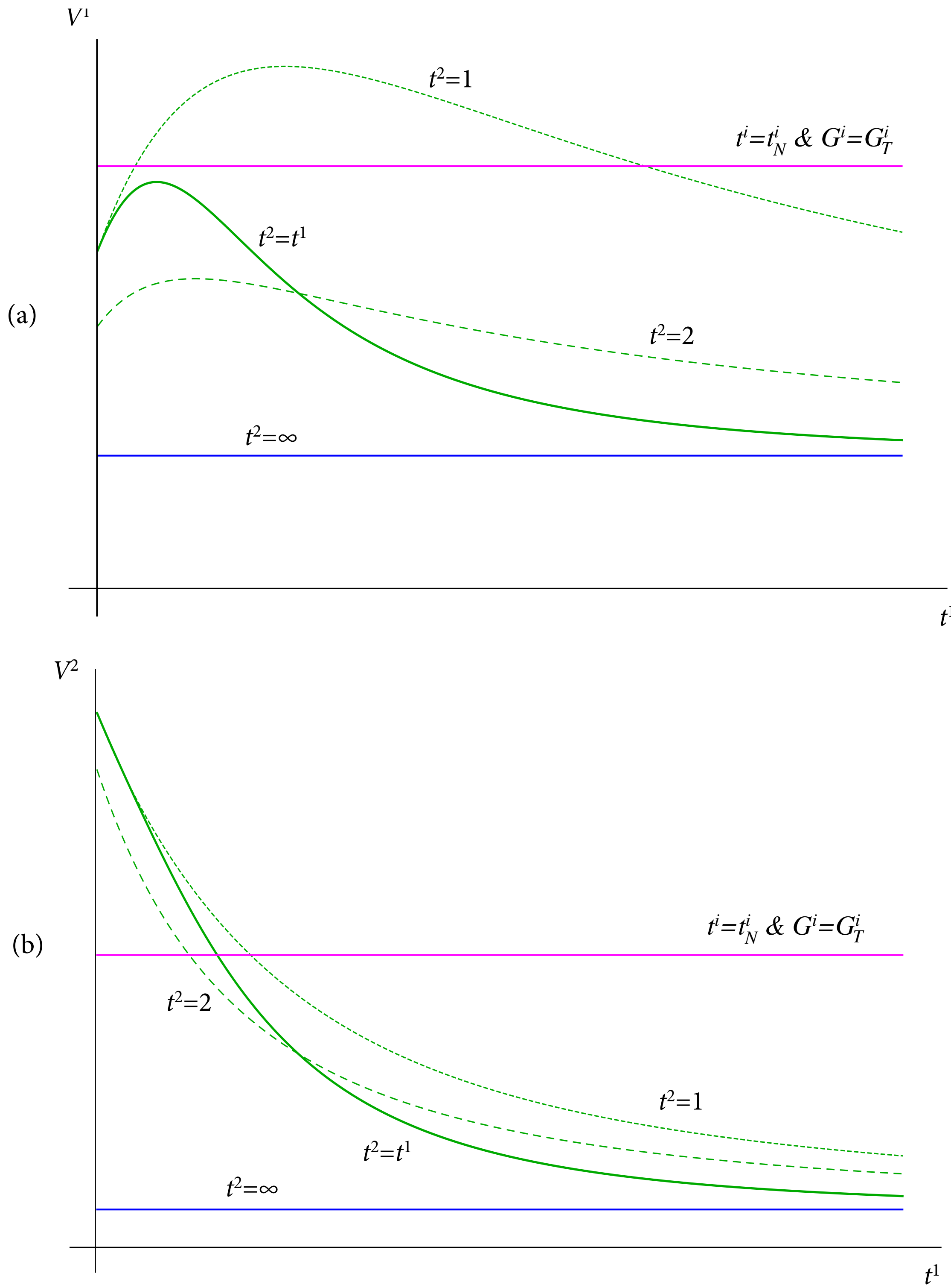


Figure B.5: The Dependence of Payoffs on Tariffs

Table B.1: Countries Included in the Sample and their ISO Codes

ISO	Country Name	ISO	Country Name
ARG	Argentina	JPN	Japan
AUS	Australia	KEN	Kenya
AUT	Austria	KOR	Korea
BEL	Belgium	KWT	Kuwait
BGR	Bulgaria	LKA	Sri Lanka
BOL	Bolivia	MAR	Morocco
BRA	Brazil	MEX	Mexico
CAN	Canada	MMR	Myanmar
CHE	Switzerland	MUS	Mauritius
CHL	Chile	MWI	Malawi
CHN	China	MYS	Malaysia
CMR	Cameroon	NER	Niger
COL	Colombia	NGA	Nigeria
CRI	Costa Rica	NLD	Netherlands
CYP	Cyprus	NOR	Norway
DEU	Germany	NPL	Nepal
DNK	Denmark	PAN	Panama
ECU	Ecuador	PHL	Philippines
EGY	Egypt	POL	Poland
ESP	Spain	PRT	Portugal
FIN	Finland	QAT	Qatar
FRA	France	ROM	Romania
GBR	United Kingdom	SEN	Senegal
GRC	Greece	SGP	Singapore
HUN	Hungary	SWE	Sweden
IDN	Indonesia	THA	Thailand
IND	India	TUN	Tunisia
IRL	Ireland	TUR	Turkey
IRN	Iran	TZA	Tanzania
ISR	Israel	URY	Uruguay
ITA	Italy	USA	United States of America
JOR	Jordan	ZAF	South Africa